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Physique du LHC et extensions du modèle standard

Par
Hamza Taibi

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Devant le jury :

Président : Ait Moussa Karim	Prof. Univ. frères Mentour-Constantine 1
Rapporteur : Habib Aissaoui	Prof. Univ. frères Mentouri-Constantine1
Examinateurs :	
Benslama Achour	Prof. Univ. fréres Mentouri-Constantine 1
Zaim Slimane	Prof. Univ. Hadj Lakhdar Batna 1
Boussahal Mounir	Prof. Univ. Msila
Moumni Mustafa	Prof. Univ. Hadj Lakhdar Batna 1

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Foremost, I thank my family, my parents to whom I dedicate this thesis.

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Chapter 1

Introduction

The standard model (SM) of particle physics is one of the biggest achievement of modern physics, together with the theory of general relativity it can explain almost all physical phenomenons from the subatomic level to the universe as a whole. The last missing piece of the SM, the Higgs boson was finally discovered in 2012 by the Atlas and CMS collaboration at the LHC [1, 2], which complete the SM picture as finalized in the 1970s. Despite the great success of the SM, there remain unanswered questions coming from some cosmological observations and inconsistencies within the model itself. Among them the hierarchy problem which includes the disparities between the masses of fermions, the weakness of gravity compared to other fundamental forces and the instability of the Higgs mass under radiative corrections, which requires unnatural fine tuning to be compatible with the data. Furthermore, astronomical observations made in the 20th century show that a big chunk of the matter of universe, necessary to explain the formation and the stability of galaxies, is constituted by invisible stuff called dark matter. Another discovery that surprised the physics community in the late 1990 is the accelerated expansion of the universe explained by the existence of mysterious energy density that act as anti gravity force expanding space and pulling galaxies apart from each other. All these observations proved the need for new theoretical framework that goes beyond the SM, models based on extra particles or extended symmetries or even extra dimension, like string theory, supersymmetry ext, were suggested to account for the short coming of the SM. An interesting class of physics beyond the standard model (BSM) is the hidden sector proposal, which describes undetectable fields interacting with SM weakly via a mediator. Our work is based on one such model, the unparticle model, proposed in 2007 by Georgi[3]. This model is formulated as an effective theory describing the interaction of a scale invariant hidden sector, unparticles, with the SM particles through contact terms. The unparticle model was not initially conceived to solve any of the challenges facing the SM. However, a lot of work have been devoted to tackle some of them, for example Kikuchi Et al in [4] formulated a model based on parity odd unparticle fields that serve as a candidate for dark matter. In the first proposal, unparticle fields were considered to be singlet under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, since then, Cacciapaglia Et al in [5] introduced unparticles that carrie quantum numbers dubbed gauged unparticle. Our work is based on that model. In this thesis we compute the unparticle contribution to electroweak observables. To reach this goal we consider unparticle fields embedded in the SM electroweak group $SU(2)_L \times U(1)_Y$. These fields would induce loop effects on electroweak precision tests represented as contribution to the oblique parameters S and T. In the course of this thesis we also tackle the problem of the muon anomalous magnetic moment(AMM) which represent a promising signal for physics beyond the SM. Recently, experiment conducted at the muon $g-2$ collaboration at Fermi Lab [6] has increased the discrepancy between the SM prediction for AMM and experimental value from 3σ deviation to 4.2σ, which is not enough to declare a discovery but it represent a strong hint for physics beyond the SM. In our work we used the spectrum of the left right symmetric model (LRSM) coming from the extra gauge bosons of the extended group $SU(2)_R$ and the Higgs sector to decrease the deviation from 2.6σ, measured at the time, to 2.5σ.

This thesis is organized as follows

The second chapter is dedicated to a review of the SM focusing on the electroweak sector relevant to our work, then we touch upon spontaneous symmetry breaking and the Higgs mechanism. Next we introduce oblique parameters which are a set of constants that parametrize the radiative oblique corrections coming from any new physics in the form of virtual particles circulating in the loops of electroweak SM gauges bosons self-energy diagrams. We present the extended version of these parameters based on the work in [7]. Finally, we give some of the challenges to the SM which motivated physicists to seek new approaches.

In chapter three we give a brief review of conformal field theory which constitutes the theoretical framework of the unparticle model

In chapter four we introduce the unparticle model as initially formulated by Georgi. Subsequent works treating conformal symmetry breaking in the unparticle sector are also examined. Next, we give a brief description of an alternative formulation to Georgi model based on spectral representation. In the last section we comment on the relation between unparticles and the

Ads/CFT correspondence.

Chapter five constitutes our original contribution. We begin by exploring the general formulation of unparticle gauged model as discussed in the literature [5, 8, 9] Then, we introduce the gauged electroweak unparticle model. We derive the vertices describing the unparticle interactions with the SM gauge bosons γ, Z and W and then we derive their asymptotic forms in the large momentum limit. Next, we use these results to compute the unparticle contribution in the scalar and fermionic cases to polarization functions of γ, Z and W, which allows us to calculate the oblique parameters S and T. The result are then used to find the region of parameter space of unparticle compatible with electroweak precision measurements. In the last section of this chapter, we estimate the scalar unparticle effects on the running of SM gauge coupling and the grand unification scale.

In chapter six we calculate the contribution of the LRSM to the muon anomalous magnetic moment which may provides an explanation for the deviation between theory and experiment.

In the last chapter we give a summary of our work and a conclusion.

Finally, we attach four appendices. In Appendix A, we present general formulas and detailed calculation of the loop integrals encountered in this manuscript. In appendix B we give an explicit computation of some constants related to unparticle vertices. In Appendix C we present the article published in mod.phys.lett A and in appendix D we present a conference paper which summerise our calculations of the muon AMM in the LRSM

Chapter 2

The standard model

The standard model (SM) is the unified theory of electromagnetism, weak and strong interactions based on three principles: gauge invariance, renormalizability and spontaneous symmetry breaking via the Higgs mechanism; We begin this review of the model by introducing the notion of gauge invariance in quantum electrodynamic.

2.1 Gauge symmetry

The first use of the concept of gauge invariance date back to the 19 century with the introduction of Maxwell equations and in the modern era it began with an observation concerning the wave equation of quantum mechanics . It has been noticed that the Schrodinger wave equation is invariant, up to a scale, under phase transformation of the type:

$$\psi \longrightarrow e^{\alpha(x)}\psi \tag{2.1}$$

$\alpha(x)$ is a time independent local parameter and the transformation (2.1) belongs to the abelian group $U(1)$. This kind of invariance is called gauge symmetry. In quantum electrodynamic, the theory must also be invariant under space time transformations to be consistent with special relativity. We must construct a lagrangian which is a Lorentz scalar as follows

$$L = i\overline{\psi}\gamma_\mu\partial^\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{2.2}$$

this lagrangian describes Free particle theory which satisfies the global symmetry of the transformation 2.1(α is a constant), but if we try to promote this global symmetry to a local one, at every point of space-time, the gauge symmetry of lagrangian (2.2) breaks down . To restore gauge invariance in this case we have to introduce a photon field A^μ with the following transformation propriety

$$A^\mu \longrightarrow A^\mu + \partial^\mu \alpha(x) \tag{2.3}$$

in order for the theory to become locally gauge invariant we replace the derivative ∂^μ in the lagrangian density (2.2) with the covariant derivative defined as

$$D^\mu = \partial^\mu + ieA^\mu \tag{2.4}$$

where e is the electric charge of the electron.

2.2 Weak interaction and gauge bosons

The first theory of the weak interaction was introduced by Fermi in 1932 [10]. It is based on contact interactions between a hadronic current and a leptonic current described by the following lagrangian

$$L_{Fermi} = -\frac{G_F}{\sqrt{2}}(\overline{p}\gamma^\mu n)(\overline{e}\gamma_\mu \nu_e) \tag{2.5}$$

G_F is the Fermi constant, n and p are the neutron and proton fields respectively . e is the electron field and ν_e is the associated neutrino field. Although, the Fermi model was successful in describing the weak interactions in β decay at the time, it can not be considered a complete theory because its not renormalizable. Moreover, the four point interaction lagrangian (2.5) violate gauge invariance. A more complete description has been proposed later by Glashow, Salam , and Weinberg [11, 12, 13] and in the process electroweak unification was discovered. This new model is based on the gauge group $SU(2) \times U(1)$. In this model the left handed components of leptons and quarks participating in the weak interaction are organized in multiplets of the group $SU(2)$ as follow

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{2.6}$$

we concentrate for simplicity on one lepton family the electron and its associated neutrino. The right handed fermions e_R and ν_L are charged under the hyper charge group $U(1)_Y$ but remain singlet under $SU(2)$. In this case the leptonic currents participating in the weak interactions are the followings

$$
\begin{aligned}
J_\mu &= 2\overline{l_L}\gamma_\mu T^+ l_L \\
J_\mu^\dagger &= 2\overline{l_L}\gamma_\mu T^- l_L \\
J_\mu^3 &= 2\overline{l_L}\gamma_\mu T^3 l_L
\end{aligned}
\tag{2.7}
$$

where $T^+ = (T_1 + iT_2)/\sqrt{2}$, $T^- = T^+ = (T_1 - iT_2)/\sqrt{2}$, and T^1 T^2, T^3 are the generators of the $SU(2)$ defined by its algebra

$$
\left[T^i, T^j\right] = i\epsilon^{ijk}T^k, \tag{2.8}
$$

with the levi-civita symbol ϵ^{ijk}. According to Neother theorem, the three currents (2.7) define three conserved charges I_1,I_2 and I_3. where I_3 is the weak isospin charge which takes different values for left handed and right handed components of the same fermionic field. For example $I_3 = \frac{1}{2}$ for e_L and $I_3 = 0$ for right handed electron e_R. Let us now gauge the weak theory, which means promoting $SU(2)_L$ to a local symmetry. For this purpose we replace, as we done in section 2.1, the field derivatives by the corresponding covariant derivatives. The weak derivative fields reads

$$
D_\mu = \partial^\mu - igT^i W_i
$$

where W_i are three $SU(2)_L$ weak bosons and g is the coupling constant of the $SU(2_L)$ group. The lagrangian density (2.5) is replaced by

$$
L = L_0 + L_c + L_n \tag{2.9}
$$

with the kinetic term

$$
L_0 = i\overline{l_L}\gamma^\mu\partial_\mu l_L + i\overline{e_R}\gamma^\mu\partial_\mu e_R + i\overline{\nu_R}\gamma^\mu\partial_\mu\nu_R
$$

and the charged interactions term

$$
L_c = gW_1^\mu\overline{l_L}\gamma^\mu T^1 l_L + gW_2^\mu\overline{l_L}\gamma^\mu T^2 l_L
$$

L_c is usually expressed in terms of the charged complex fields $W_\mu^\pm = \left(W_\mu^1 \pm W_\mu^2\right)/\sqrt{2}$. The last term in the lagrangian (2.9) is

$$
L_n = gW_3^\mu\overline{l_L}\gamma^\mu T^3 l_L
$$

this term corresponds to a neutral interaction which does not exist in the classical Fermi theory, at the same time it can't be identified with the electromagnetic interaction because parity is conserved in this interaction, left handed and right handed component of a fermionic field participate equally in the electromagnetic interaction, which is not the case for weak interactions. The simplest solution to include electromagnetic interaction in this construct is to extend the gauge group $SU(2)_L$ to include the abelien group $U(1)_Y$.The lagrangian density must be invariant under $U(1)_Y$ gauge transformations of the types(2.1), introduced in section 2.1. In this case however, the generator of this transformation is the hypercharge operator Y related to the charge operator Q through the Gellman-Nishijima formula

$$T^3 + \frac{Y}{2} = Q$$

in this case a gauge field B^μ corresponding to the hypercharge group $U(1)_Y$ must be introduced and the covariant derivative of the extended group $SU(2)_L \times U(1)_Y$ takes the form

$$D^\mu = \partial^\mu - igT^iW_i - ig'\frac{Y}{2}B^\mu \tag{2.10}$$

g' is the coupling constant of the group $U(1)_Y$. Replacing D^μ from eq(2.10) in the lagrangian density (2.9) we find the neutral interaction term

$$L_n = gW_3^\mu \overline{l_L}\gamma^\mu T^3 l_L + \tfrac{g'}{2}B^\mu \left(\overline{l_L}\gamma^\mu Y l_L + Y(e_R)\overline{e_R}\gamma^\mu e_R + Y(\nu_R)\overline{\nu_R}\gamma^\mu \nu_R\right) \tag{2.11}$$

where Y is the hypercharge operator in the doublet representations of the $SU(2)$ group, $Y(e_R)$ is the hypercharge number of the right handed electron and $Y(\nu_R)$ is the hypercharge number for right handed neutrino. Now we define the column vector

$$\psi = \begin{pmatrix} \nu_L + \nu_R \\ e_L + e_R \end{pmatrix} \tag{2.12}$$

we can rewrite the eq(2.11) as follows

$$L_n = \overline{\psi}\gamma^\mu \left(gT^3 W_3^\mu + g'\frac{Y}{2}B^\mu \right) \psi \tag{2.13}$$

to identify the part responsible for electromagnetic interactions and the part involved in neutral weak interactions, we perform a rotation in the internal isospin space that transform the weak eigenstates W_3^μ and B^μ to (physical) eigenstates Z^μ and A^μ as follow

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos(\theta_W) & \sin(\theta_W) \\ -\sin(\theta_W) & cos(\theta_W) \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} \tag{2.14}$$

where the angle θ_W is the Weinberg (electroweak) mixing angle defined as

$$sin^2(\theta_W) = \frac{g'^2}{g^2 + g'^2} \simeq 0.23$$

Now substituting the vector fields W_3^μ and B^μ in eq(2.13) by the mixed states (2.14) we find

$$\begin{aligned} L_n =& \overline{\psi}\gamma^\mu \left(g\cos(\theta_W)T_3 - \frac{Y}{2}g'\sin(\theta_W) \right) \psi Z^\mu \\ &+ \overline{\psi}\gamma^\mu \left(g\sin(\theta_W)T_3 + \frac{Y}{2}g'\cos(\theta_W) \right) \psi A^\mu \end{aligned} \tag{2.15}$$

with the appropriate choices of hypercharge numbers for left and right handed leptons, $Y(l_L)$,$Y(e_R)$ and $Y(\nu_R)$, we can make the identification

$$eQ = g\sin(\theta_W)T_3 + \frac{Y}{2}g'\cos(\theta_W) \tag{2.16}$$

by comparing this expression with the Gellman formula $Q = T_3 + \frac{Y}{2}$ we deduce the following relation

$$g\sin(\theta_W) = g'\sin(\theta_W) = e$$

The first term in eq(2.15) defines the weak neutral current interaction with the weak neutral vector boson Z^μ, by comparison to (2.16) we can deduce the corresponding charge operator for weak neutral interactions

$$Q_Z = \frac{1}{\sin(\theta_W)\cos(\theta_W)} \left(T_3 - Q\sin^2(\theta_W) \right)$$

So far we have constructed a gauge invariant theory of weak and electromagnetic interactions. These interactions are carried out by four massless vector gauge bosons. In reality however, there is only one massless vector bosons, the photon, associated with the infinite range of electromagnetic interaction. To make the standard model consistent with observation we must introduce mass terms for the other weak gauge bosons in a subtle way, without destroying the gauge symmetry of the theory. This task has been achieved by Petter Higgs and Other with the Higgs mechanism.

2.3 The Higgs mechanism

Spontaneous symmetry breaking is the process in which a symmetry exhibited by a theory is not manifested in the solutions of that theory. Applying this process to local gauge theories, like the electroweak standard model, allow as to give masses to particles without spoiling the gauge invariance of the model necessary for the renormalizability of the theory. This method is called the Higgs mechanism. In this mechanism massless fermions and bosons of the SM acquire mass via the interaction with universal scalar field called the Higgs field. The fluctuation of this field around a minimum ground state behave like a particle of well defined mass called the Higgs boson. To apply the Higgs mechanism [14, 15] to the SM we introduce a scalar doublet of the $SU(2)$ group

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \tag{2.17}$$

where the upper component is a charged complex field and the lower component is a neutral complex field. The lagrangian for this scalar field is the following

$$L = D^\mu \Phi^\dagger D_\mu \Phi + V(\Phi)$$

D^μ is the covariant derivative defined in eq(2.10) and the Higgs potential V is chosen to be of the form

$$V(\Phi^\dagger \Phi) = \lambda(\Phi^\dagger \Phi)^2 - \mu^2 \Phi^\dagger \Phi \tag{2.18}$$

λ is a positive parameter. If $\mu^2 < 0$, then V accept a trivial minimum at $\Phi = 0$. If in the other hand $\mu^2 > 0$, V has a minimum at

$$\mid \Phi \mid = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}} \tag{2.19}$$

so we have a degenerate minimum as depicted in Fig 2.1. Now we break the symmetry by choosing a particular direction in the internal $SU(2)$ space for the minimum (2.19)

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.20}$$

this particular choice leaves the residual $U(1)_{em}$ invariance intact because the transformations generated by the charge operator $Q = T^3 + Y$ leave the vacuum expectation value (2.19) invariant

$$Q\Phi = 0$$

So we have three broken generators (T_1,T_2,T_3) and one unbroken generator Q. According to Goldstone theorem [16, 17], for each broken generator of the

$SU(2)$ group T_i, there is a massless gauge boson $\theta^i(x)$ called the Goldstone boson. Therefore, the fluctuating field around the minimum 2.19 takes the following form

$$\Phi = \exp\big(iT_i\theta^i(x)\big)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix} \tag{2.21}$$

h is real scalar field called the Higgs boson. To get read off the unphysical Goldstone bosons we perform a gauge transformation in the internal $SU(2)$ space as follow

$$\Phi \to U(\theta)\Phi \tag{2.22}$$

$$T^i W_i^\mu \to UT^i W_i^\mu U^{-1} + \frac{i}{g}(\partial^\mu U)U^-1 \tag{2.23}$$

Substituting the new field $\Phi' = U(\theta)\Phi$ in L with the usual covariant derivative $D^\mu = \partial^\mu - igT^iW_i^\mu - ig'B^\mu\frac{Y}{2}$, we find

$$\begin{aligned} L = \; & \left(\partial^\mu + igT^iW_i^\mu - ig'B^\mu\tfrac{Y}{2}\right)\Phi^\dagger\left(\partial^\mu - igT^iW_i^\mu - ig'B^\mu\tfrac{Y}{2}\right)\Phi \\ & +\mu^2(v+h)^2 - \lambda((v+h)^2)^2 \end{aligned} \tag{2.24}$$

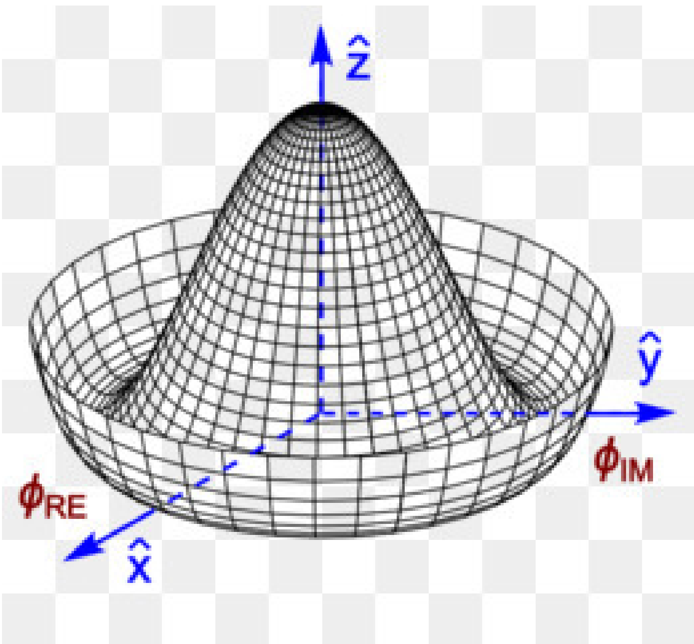

Figure 2.1: Higgs potential.

Expanding the lagrangian we can extract the term quadratic in the vector boson fields

$$L_M = \frac{v^2}{8}\left[\left(gW^3_\mu - g'B_\mu\right)\left(gW^{3\mu} - g'B^\mu\right) + 2g^2 W^-_\mu W^{+\mu}\right]$$

the last term is the mass term of the charged vector bosons W^+,W^- with

$$M_W = \frac{v}{2}g$$

to find the mass term for the neutral vector boson Z we perform the same rotation (2.14) done in the last section from weak eigenstates W^3_μ,B^μ to mass eigenstates Z^μ and A^μ. After this rotation the quadratic term in vector bosons fields become

$$L_M = \frac{g^2v^2}{4}W^-_\mu W^{+\mu} + \frac{(g^2+g'^2)v^2}{8}Z_\mu Z^\mu$$

we find that the Z boson has acquired mass given by

$$M_Z = \frac{v\sqrt{g^2+g'^2}}{2}$$

Using the relation $g\sin(\theta_W) = g'\cos(\theta_W)$ we find

$$M_Z = \frac{M_W}{\cos(\theta_W)}$$

the absence of a mass term for A^μ means that the photon remain massless as expected. The three massless Goldstone bosons have been eaten up to generate masses for the vector bosons. There remains one scalar boson the Higgs particle. To find its mass we examine the Higgs potential(2.18), after spontaneous symmetry breaking, we find the following result

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda}v$$

until 2012 the Higgs mass remained a free parameter of the SM. After the discovery of the Higgs boson at Atlas and CMS collaborations [1, 2] the Higgs mass was found to be

$$M_h \simeq 125\ Gev$$

2.4 Yukawa terms

In the previews section, we used the Higgs mechanism to generate masses for vector gauge bosons without destroying the symmetry of the lagrangian, but fermions remain massless. The introduction of a mass term in the fermionic lagrangian is not allowed in electroweak theory because the mass term $m\overline{\psi}_L\psi_R$ violate electroweak gauge invariance(left handed fermions and right handed fermions have different hypercharge numbers). Fortunately, we can use the same mechanism, spontaneous symmetry breaking, to give masses to fermions via the interaction with the Higgs field as follows

$$L_{Yukawa} = -Y_e\overline{l_L}\phi e_R - Y_d\overline{Q}\phi d_R - Y_u\overline{Q}iT_2\phi^* u_R$$

here we have written Yukawa terms for the first family of leptons and quarks for simplicity. When the Higgs field acquires non vanishing expectation value (vev), the fermions gain masses. Then, the Yukawa lagrangian becomes

$$\begin{aligned} L_{Yukawa} = \quad & -\left(\frac{Y_e v}{\sqrt{2}}\right)\bar{e}e - \left(\frac{Y_d v}{\sqrt{2}}\right)\bar{d}d - \left(\frac{Y_u v}{\sqrt{2}}\right)\bar{u}u \\ & -\left(\frac{Y_e}{\sqrt{2}}\right)\bar{e}he - \left(\frac{Y_d}{\sqrt{2}}\right)\bar{d}hd - \left(\frac{Y_u}{\sqrt{2}}\right)\bar{u}hu \end{aligned} \tag{2.25}$$

from the first line we can deduce the masses of the fermions

$$\begin{aligned} m_e &= \frac{Y_e v}{\sqrt{2}} \\ m_u &= \frac{Y_u v}{\sqrt{2}} \\ m_d &= \frac{Y_d v}{\sqrt{2}} \end{aligned} \tag{2.26}$$

the second line in eq(2.25) corresponds to the Higgs-fermions couplings. We notice that the coupling of Higgs bosons to fermions is proportional to their masses

$$g_{f,h} = \frac{\sqrt{2}}{v} m_f$$

2.5 Oblique parameters

Despite the success of the SM there remains unsolved puzzles like the hierarchy problem, related to the instability of the Higgs mass under radiative corrections, or the unknown nature of dark matter, which constitutes 75% of the matter of the universe. A lot of new physics models, which involve new particles or extended symmetries, have been proposed to solve these puzzles. However, so far there is no direct evidence of physics beyond the SM. So, we have to rely on all sources of indirect informations that current measurements provide. The most important of these measurements are precision study of electroweak observables. These are the study of radiative corrections to scattering amplitudes of light fermions involved in electroweak interactions like for example $ee \to qq$ mediated by a photon or a Z boson. The most significant of these radiative corrections are self-energy corrections to gauge bosons propagators known as oblique corrections (see Fig (2.2)). Since the internal states running in the loops can be arbitrarily heavy, new models can be tested by computing their contributions to electroweak observables via oblique corrections and comparing the new results to experimental constraints on these observables, which allow theorists to constraint their models. The effects of oblique corrections on fermions scattering can be determined by examining how the vacuum polarization functions

$$\Pi^{\mu\nu}_{ab} = \Pi_{ab}(q^2) g^{\mu\nu} + (q^\mu q^\nu term)$$

with $a, b = \gamma, Z, W$ appear in the new physics contribution to electroweak observables. Since the most precisely measured quantities are probed at specific momentum transfer scales, namely at low energy experiments $q^2 = 0$ and at the Z resonance $q^2 = M_Z$, and $q^2 = M_W$, all oblique corrections can be parametrized by a limited number of independent parameters. The first three S, T, and U were introduced by Peskin and Takeuchi [18, 19] under the assumptions that the new physics states are mush heavier than the gauge bosons masses $\frac{M_Z}{M} \ll 1$. In this case the polarization functions can be expended to first order as follows

$$\Pi_{ab} = A_{ab} + q^2 B_{ab} \tag{2.27}$$

where $A_{ab} = \Pi_{ab}(0)$ and $B_{ab} = \Pi'_{ab} = \dfrac{d\Pi_{ab}(q^2)}{dq^2}$. Since we have four polarization functions $\Pi_{\gamma\gamma}$, Π_{ZZ}, $\Pi_{Z\gamma}$, and Π_{WW}, the expansion Eq(2.27) defines eight unknown quantities. Because of electromagnetic gauge invariance we have

$$\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0$$

thus, we are left with six independent parameters. We can eliminate another three parameters by using the most precisely measured electroweak observables and fitting them with three SM parameters, those are the fine structure constant α, measured from low energy scattering experiments, and G_F as measured from muon decay, and M_Z. So, we have finally three independent linear combinations of the $\Pi_{ab}s$ functions which are S, T, and U defined as

$$S = \frac{4 s_w^2 c_w^2}{\alpha} \left(\frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{M_Z^2} - \frac{c_w^2 - s_w^2}{s_w c_w} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right) \tag{2.28}$$

$$T = \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right] \tag{2.29}$$

and

$$U = \frac{4s_w^2}{\alpha}\left(\frac{\Pi_{WW}(M_{WW}^2) - \Pi_{WW}(0)}{M_{WW}^2} - c_w^2\left(\frac{\Pi_{ZZ}(M_{ZZ}^2) - \Pi_{ZZ}(0)}{M_{ZZ}^2}\right)\right.$$
$$\left. - s_w^2\Pi'_{\gamma\gamma}(0) - 2s_wc_w\Pi'_{Z\gamma}(0)\right) \tag{2.30}$$

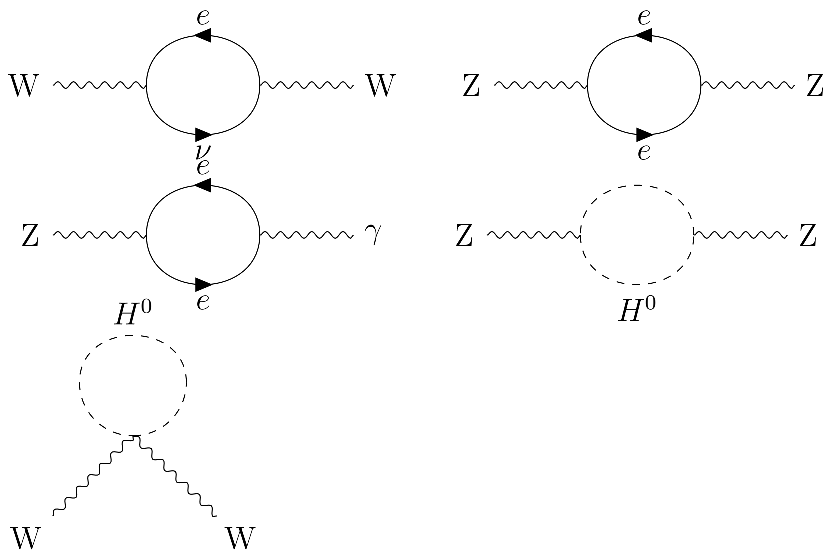

Figure 2.2: Examples of Feynman diagrams contributing to oblique corrections.

each of the these three quantities has physical meaning. S quantifies the difference in mixing between the hypercharge and the third weak isospin current at $q^2 = M_Z^2$ and $q^2 = 0$. T describes the amount of custodial symmetry breaking at $q^2 = 0$. On the other hand U measure the contribution of the W, Z mass non-degeneracy to weak isospin breaking. As mentioned above, the STU formalism is based on the assumptions that BSM physics enter at a scale far above the weak scale ($M >= 1Tev$). However, when the new physics scale is not large in comparison to electroweak breaking scale, the linear approximation (2.27) becomes inaccurate. In ref [7], the authors extend the approximation (2.27) to include quadratic terms in q^2. This extra term necessitates the introduction of another 3 oblique parameters called V, W and X that vanish in the linear approximation

$$\alpha V = \Pi'_{ZZ}(M_Z^2) - \left(\frac{\Pi'_{ZZ}(M_Z^2) - \Pi'_{ZZ}(0)}{M_Z^2}\right)$$

Expressions for observables
$\Gamma_Z = (\Gamma_Z)_{SM} 0.00961S + 0.0263T + 0.0194V 0.0207X (GeV)$
$\Gamma_{bb} = (\Gamma_{bb})_{SM} 0.00171S + 0.00416T + 0.00295V 0.00369X (GeV)$
$\Gamma_{l^+l^-} = (\Gamma_{l^+l^-})_{SM} 0.000192S + 0.000790T + 0.000653V 0.000416X (GeV)$
$\Gamma_{had} = (\Gamma_{had})_S M 0.00901S + 0.0200T + 0.0136V 0.0195X (GeV)$
$A_{FB}(\mu) = (A_{FB}(\mu))_{SM} 0.00677S + 0.00479T 0.0146X$
$A_{pol}(\tau) = (A_{pol}(\tau))_{SM} 0.0284S + 0.0201T 0.0613X$
$A_e(P_\tau) = (A_e(P_\tau))_{SM} 0.0284S + 0.0201T 0.0613X$
$A_{FB}(b) = (A_{FB}(b)_{SM} 0.0188S + 0.0131T 0.0406X$
$A_{FB}(c) = (A_{FB}(c)_{SM} 0.0147S + 0.0104T 0.03175X$
$A_{LR} = (A_{LR})_{SM} 0.0284S + 0.0201T 0.0613X$
$M_W^2 = (M_W^2)_{SM} \, (1 - 0.00723S + 0.0111T + 0.00849U)$
$\Gamma_W = (\Gamma_W)_{SM} \, (10.00723S - 0.0333T + 0.0849U 0.0781W (GeV)$
$g_L^2 = (g_L^2)_{SM} - 0.00269S + 0.00663T$
$g_R^2 = (g_R^2)_{SM} + 0.00937S - 0.000192T$
$g_V^e(\nu e \to \nu e) = (g_V^e)_{SM} + 0.00723S - 0.00541T$
$g_A^e(\nu e \to \nu e) = (g_A^e)_{SM} + -0.00395T$
$Q_W({}^{133}_{55}\mathrm{C_s}) = Q_W(C_s)_{SM} - 0.795S - 0.0116T$

Table 2.1: Summary of the dependence of electroweak observables on S,T,U,V,W and X.

$$\alpha W = \Pi'_{WW}(M_W^2) - \left(\frac{\Pi'_{WW}(M_W^2) - \Pi'_{WW}(0)}{M_W^2} \right)$$

$$\alpha X = -s_w c_w \left(\frac{\Pi'_{Z\gamma}(M_Z^2)}{M_Z^2} - \Pi'_{Z\gamma}(0) \right)$$

to obtain bounds on new physics contribution, we have first to calculate oblique corrections in the SM to various electroweak observables and then express the same observables in terms of the oblique parameters S through X, an example is given in Table 2.1. Extracting the SM model predictions for the most precisely measured quantities from experimental results allow us to make a global fit and find constraints on oblique parameters. After the discovery of the Higgs boson with mass $m_h \simeq 125$, the reference point for SM calculations has shifted. For the oblique parameters S, T, and U the best bounds found so far are the following [20]

$$\begin{aligned} \Delta S &= S - S_{SM} = 0.05 \pm 0.11 \\ \Delta T &= T - T_{SM} = 0.09 \pm 0.13 \\ \Delta U &= U - U_{SM} = 0.09 \pm 0.094 \end{aligned} \tag{2.31}$$

2.6 SM challenges

Despite the great success of the SM in describing almost all experimental data, it fails to explain some phenomenons which indicates the need for new physics above the electroweak scale. In this section we give a short summery of some of the challenges facing the SM today

2.6.1 The hierarchy problem

The discovery of the SM at LHC with mass $M_h \simeq 125$ completed the SM picture. However, there is a big discrepancy between the effective value (experimental value) and the theoretical prediction of the Higgs mass if we include radiative corrections, of the type depicted in fig 2.3, quadratic corrections contribute a term proportional to the ultraviolet energy scale Λ

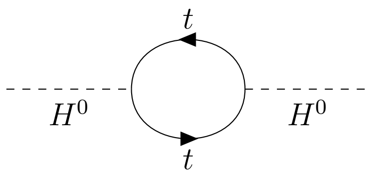

Figure 2.3: top quark contribution to Higgs self energy.

$$\Delta m_H = -\frac{\lambda_f^2}{8\pi^2}\left[\Lambda_{UV}^2 + \dots\right] \tag{2.32}$$

If we suppose that the SM is a valid description of nature up to the plank scale, the Higgs mass will gain an enormous contribution from Λ, which contradict experiment, In order to solve this problem we have to add a counter term to the lagrangian fine tuned to 10^{-30} which is in contradiction with the principle of naturalness. This problem find solutions in new physics proposal like supersymmetry [21], where for each fermion there is bosonic partner contributing to the Higgs mass with similar term like (2.32), but with the opposite sign (see Fig(2.4))

$$\Delta m_H = \frac{\lambda_S^2}{8\pi^2}\left[\Lambda_{UV}^2 + \dots\right]$$

this make the total contribution, from radiative corrections, to the Higgs mass equal to zero if we include both the fermionic and bosonic particles (here we take $\lambda_S = \lambda_f$).

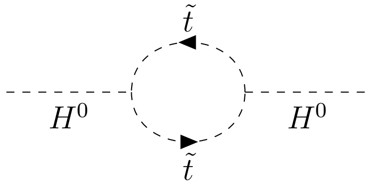

Figure 2.4: top super-partner contribution to the Higgs self energy.

2.6.2 Dark matter and Dark energy

Astronomical observations since the 1930th have found that 75% of the matter in the universe [22], responsible for holding galaxies together is not composed of ordinary matter, that is to say, it is not composed by particles predicted by the SM. A lot of proposed models including supersymmetry have candidate particles to explain the nature of dark mature. The most popular ones are the so called WIMP particles proposals which mean weakly interacting matter particles. In addition to dark matter the discovery of the accelerated expansion of the universe in the late 1990 [23] indicates the existence of a mysterious energy with constant density, identified with the cosmological constant of Einstein theory of general relativity. This so called dark energy does not have any explanation in the SM.

2.6.3 Neutrino oscillations

Experiments performed at the Super-Kamiokade Observatory in Japan [24] and others have shown that solar neutrino change flavour while travelling from the sun to particle detectors at Earth. This phenomenon known as neutrino oscillations indicates that neutrino have a non vanishing mass which contradict the SM. We can solve this problem by adding right handed neutrinos in the model or by allowing a Majorana mass term for neutrinos. These proposals do not explain the disparity between the measured neutrino mass which is very tiny in comparison to other leptons, which means that this problem does not yet have a complete solution.

Chapter 3

Conformal field theory

3.1 Conformal transformations

Consider a spacetime of dimension D endowed with the metric tensor $\eta_{\mu\nu}(-1, 1, ..., 1)$. A conformal transformation is the subgroup of coordinate transformations that leave the metric invariant up to a scale

$$\eta'_{\mu\nu} = \Omega(x)\eta_{\mu\nu}$$

a subset of this transformation with $\Omega(x) = 1$ is the Poincaré group which preserves distances. Conformal transformations (CT) leave the angles between two arbitrary vectors or curves invariant but change distances. To determine the set of parameters of the conformal group, we consider the infinitesimal transformation

$$x'_\mu = x_\mu + \epsilon_\mu \tag{3.1}$$

under a general coordinate transformation the metric tensor takes the form

$$\eta'_{\mu\nu} = \frac{\partial x'_\mu}{\partial x_\rho}\frac{\partial x'_\nu}{\partial x_\sigma}\eta_{\rho\sigma} \tag{3.2}$$

substituting (3.1) in (3.2) and using a first order approximation we find

$$\eta'_{\mu\nu} = \eta_{\mu\nu} - (\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) + o(\epsilon^2)$$

the requirement that the transformation be conformal implies that

$$\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu = f(x)\eta_{\mu\nu} \tag{3.3}$$

where $f(x)$ is some function. If we take the trace on both sides of (3.3) we find

$$f(x) = \frac{2}{D}\partial_\rho\epsilon^\rho \tag{3.4}$$

By applying an extra derivatives on eq (3.3), permuting the indices, contracting with $\eta_{\mu\nu}$ we arrive at the following equation

$$(D-1)\,\partial^2 f = 0$$

If $D = 1$ there is no constraints on the function f which is not surprising since in one dimension there is no angles. The case $D = 2$ is a special case that we will not consider in this thesis. For $D \geq 3$, we see that $f(x)$ is at most linear in x, It follows from eq(3.3) that ϵ_μ is at most quadratic in x. So this set of transformations involves a finite dimensional conformal group which takes the general form

$$\epsilon^\mu = a^\mu + M^{\mu\nu}x_\nu - \lambda x^\mu + b_\nu \left(2x^\mu x^\nu - \eta^{\mu\nu}x^2\right)$$

each parameter of this equation corresponds to an infinitesimal transformation as follows

$$\begin{array}{cc}
\text{Translations} & x'^\mu = x^\mu + a^\mu \\
\text{dilation:} & x'^\mu = \lambda x^\mu \\
\text{rotation:} & x'^\mu = M^\mu_\nu x^\nu \\
\text{SCT} & \text{x}'^\mu = \dfrac{x^\mu - b^\mu x^2}{1 - 2b.x + b^2x^2}
\end{array}$$

the generators corresponding to translation and rotations are the momentum vector P^μ and the angular momentum tensor $L^{\mu\nu}$ respectively. The dilatations parametrized by λ is generated by the operator D, the special conformal transformation (SCT) is represented by K^μ. These generators obey the following commutation relations, which define the conformal algebra

$$\begin{aligned}
[\,D,\, P^\mu\,] &= iP^\mu \\
[\,D,\, K^\mu\,] &= iK^\mu \\
[\,K^\mu,\, P^\nu\,] &= 2i\left(\eta^{\mu\nu}D - L^{\mu\nu}\right) \\
[\,K^\rho,\, L^{\mu\nu}\,] &= i\left(\eta^{\rho\mu}K^\nu - \eta^{\rho\nu}K^\mu\right)
\end{aligned} \tag{3.5}$$

in addition to the familiar Poincaré algebra

$$\begin{aligned}
[\,P^\rho,\, L^{\mu\nu}\,] &= i\left(\eta^{\rho\mu}P^\nu - \eta^{\rho\nu}P^\mu\right) \\
[\,L^{\mu\nu},\, L^{\rho\sigma}\,] &= i\left(\eta^{\nu\rho}L^{\mu\sigma} + \eta^{\mu\sigma}L^{\nu\rho} - \eta^{\mu\rho}L^{\nu\sigma} - \eta^{\nu\sigma}L^{\nu\rho}\right) \\
[\,P^\mu,\, P^\nu\,] &= 0.
\end{aligned} \tag{3.6}$$

From the generators mentioned above we can count the number of generators of the conformal group explicitly as

$$\begin{aligned}
&1\ dilatation + D\ translations + \frac{D(D-1)}{2}\ rotations \\
&= \frac{(D+1)\,(D+2)}{2}\ generators
\end{aligned} \tag{3.7}$$

This is exactly the number of generators of the $SO(D,2)$ group. To make this point more explicitly let us redefine the operators of eq(3.5) and eq(3.6) as follows

$$J_{\mu,\nu} = L_{\mu\nu}, J_{-1,\mu} = \frac{1}{2}\left(P_\mu - K_\mu\right), J_{0,\mu} = \frac{1}{2}\left(P_\mu + K_\mu\right), j_{-1,0} = D$$
$$for\ J_{ab} = -J_{ba}\ and\ a,b \in \{-1,-1,...,D\} \tag{3.8}$$

Then the generators J_{ab} can be shown to obey the $SO(D,2)$ commutation relations

$$[\ J_{ab},\ J_{cd}\ ] = i\left(\eta_{ad}J_{bc} + \eta_{bc}J_{ad} - \eta_{ac}J_{bd} - \eta_{bd}J_{ac}\right) \tag{3.9}$$

with the metric $\eta\,(-1,1,...,1)$

3.2 Representations of the conformal group

In order to find the representations of the conformal group in D dimension, we have to show how classical fields are affected by conformal transformations. For this purpose we consider an infinitesimal conformal transformations parametrized by ω_g. Our goal is to find a matrix representations such that the field $\phi(x)$ transform as

$$\phi'(x') = (1 - i\omega_g T_g)\phi(x)$$

the method used in this case is to consider the action of the little group, which is the subgroup that leaves the origin ($x = 0$) invariant. For the Poincaré group the little group is identified as the Lorentz group. To find matrix representations for the Lorentz group we use the following transformation formula

$$L^{'}_{\mu\nu}(x)\phi(x) = e^{-iP^\lambda x_\lambda} L_{\mu\nu}(0) e^{iP^\lambda x_\lambda}$$

Using the Haudssauf formula

$$e^{-A}Be^{A} = B + [\ B, A\ ] + ...$$

and the commutation relations (5.56) of the Poincaré group we find

$$L^{'}_{\mu\nu}(x)\phi(x) = e^{-iP^\lambda x_\lambda} L_{\mu\nu}(0) e^{iP^\lambda x_\lambda} = L_{\mu\nu}(0) + x_\mu P_\nu - x_\nu P_\mu$$

which gives the following

$$L^{'}_{\mu\nu}(x)\phi(x) = i\left(x_\nu \partial_\mu - x_\mu \partial_\nu\right) + S_{\mu\nu}\phi(x)$$

where $S_{\mu\nu}$ is the spin matrix of the field $\phi(x)$ and $i\partial_\mu$ is the familiar momentum operator in configuration space. Now in the same manner we consider the full conformal group. We consider the groups of rotations, dilatations and SCT which leave $x = 0$ invariant. We denote them by $S_{\mu\nu}$, $\tilde{\triangle}$ and κ_μ respectively . These matrix representation must satisfy the reduced algebra of the conformal group

$$\begin{aligned}
&[\, \tilde{\triangle},\, S_{\mu\nu}\,] = 0\\
&[\, \tilde{\triangle},\, \kappa_\mu\,] = -i\kappa\mu\\
&[\, \kappa_\mu,\, \kappa_\nu\,] = 0\\
&[\, \kappa_\rho,\, S_{\mu\nu}\,] = i\left(\eta_{\rho\mu}\kappa_\nu - \eta_{\rho\nu}\kappa_\mu\right)\\
&[\, S_{\mu\nu},\, S_{\rho\sigma}\,] = i\left(\eta_{\nu\rho}S_{\mu\sigma} + \eta_{\mu\sigma}S_{\nu\rho} - \eta_{\mu\rho}S_{\nu\sigma} - \eta_{\nu\sigma}S_{\mu\rho}\right)
\end{aligned} \tag{3.10}$$

Following similar steps as before, we find

$$\begin{aligned}
&e^{iP_\lambda x^\lambda} D e^{-iP^\lambda x_\lambda} = D + x^\mu P_\mu\\
&e^{iP_\lambda x^\lambda} K_\mu e^{-iP^\lambda x_\lambda} = K_\mu + 2x_\mu D - 2x^\nu L_{\mu\nu} + 2x_\mu\left(x^\lambda P_\lambda\right) - x^2 P_\mu
\end{aligned} \tag{3.11}$$

Now if we consider a classical field ϕ that belongs to the irreducible representation of the Lorentz group, according to Shur lemma, an operator that commute with all the generators $S_{\mu\nu}$ is proportional to the identity matrix. So, $\tilde{\triangle} = i\triangle$ where $\triangle$ is a number which is called the scaling dimension of the field $\phi(x)$. The inclusion of imaginary number i is due to the fact that the representation of the dilatation group on classical fields are non unitary. In a classically scale invariant theory(for example a free field theory) the scale dimension coincide with the canonical dimension $d_O = \triangle[O]$ of an operator O. However, in a renormalized quantum field theory, renormalization effects, introduce a scale into the theory which spoil scale invariance. In this case $d_O \neq\ \triangle[O]$, the difference $\gamma = d - \triangle$ is called the anomalous dimension of the operator O. Finally, since $\tilde{\triangle}$ is proportional to the identity, using the commutations rules (3.10)we conclude that the κ matrices vanish. This gives us the following transformations rules for the field ϕ

$$\begin{aligned}
&P_\mu\phi(x) = -i\partial_\mu\phi(x)\\
&L_{\mu\nu}\phi(x) = i\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right)\phi(x) + S_{\mu\nu}\phi(x)\\
&D\phi(x) = -i\left(x^\mu\partial_\mu + \triangle\right)\phi(x)\\
&K_\mu\phi(x) = \left(-2i\triangle x_\mu - x^\nu S_{\mu\nu} - 2ix_\mu x^\nu\partial_\nu + ix^2\partial_\mu\right)\phi(x)
\end{aligned} \tag{3.12}$$

From the above results, we can in principle derive the transformation rules for $\phi(x)$ under a finite conformal transformation. We will only consider the result for spineless fields($S_{\mu\nu} = 0$). The transformations rule is given by

$$\phi(x) \to \phi'(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\triangle/D} \phi(x) \tag{3.13}$$

Fields transforming in this manner are called quasi-primary fields

3.3 Primary and descendant operators

In section 3.6 we will compute the general form of correlation functions for primary operators using their transformation proprieties under the conformal group. In this section, we will present these transformations determined by the representation theory of the conformal group.

Following Mack and Salam [25], by restricting our study to operators inserted at $x = 0$, we can find proprieties of operators inserted in other locations by applying a translation

$$O(x) = e^{x.p} O(0) e^{-x.p} \tag{3.14}$$

we will take $O(0) = O^i_{\Delta,r}(0)$ to be the finite dimensional irreducible representation r of the rotation group. These operators are characterized by their scaling dimension Δ which is the eigenvalue of the dilatation operator D:

$$\begin{aligned} &[D, O^i_{\Delta,r}(0)\,] = \Delta O^i_{\Delta,r}(0) \\ &[L_{\mu\nu}, O^i_{\Delta,r}(0)\,] = (R_{\mu\nu})^i_j\, O^i_{\Delta,r}(0) \end{aligned} \tag{3.15}$$

where $R_{\mu\nu}$ are the generators of the representation r of the $SO(D)$ group. The operators P^μ and K^μ act on D as raising and lowering operators respectively, which allow us to construct the eigen-fields (spectrum) of the dilation operator. However, in any physically interesting theory, the spectrum of the dilation operator is real and bounded from below. So, the conformal multiplet must contain an operator of lowest dimension:

$$[K^\mu, O^i_{\Delta,r}(0)\,] = 0 \tag{3.16}$$

operators satisfying this condition are called primary operators. The other operators of the conformal spectrum are called descendants and they are obtained from primary operators by acting on them with P^μ, in other words, the descendent operators are derivatives of the primary operators.

The Operator $O(0)$ is characterized by two main quantum numbers, its scaling dimension Δ and its irreducible representation under the rotation group. It is important to know the transformation proprieties of any operator $O(x)$ under general conformal transformation which can be achieved using equations (3.14.3.15) here we will give its the explicit form

$$O'_{\Delta,r}{}^i(x') = \mathcal{F}^i_j O^i_{\Delta,r}(x),\ \mathcal{F} = \frac{1}{(\Omega(x))^\Delta} R(M^\mu_\nu(x)) \tag{3.17}$$

where $R(M^\mu_\nu(x))$ is the matrix representation of the finite rotation M^μ_ν in the representation r.

3.4 The Energy-Momentum Tensor

According to Neother theorem every continuous symmetry implies the existence of a conserved current and a conserved charge. Under translation the associated current is the energy-momentum tensor defined by

$$T_C^{\mu\nu} = -\eta^{\mu\nu}\mathcal{L} + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)} \tag{3.18}$$

$T_C^{\mu\nu}$ is the canonical energy-momentum tensor. The general expression for a conserved current associated with the infinitesimal coordinate transformations $x^{'\mu} = x^\mu + \epsilon^\mu$ and the corresponding field transformation $\phi^{'}(x^{'}) = \mathcal{F}\phi(x)$ is

$$J^\mu = \left[\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)}\partial_\nu \phi - \delta_\nu^\mu \mathcal{L}\right]\frac{\delta x^\nu}{\delta \epsilon} - \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)}\frac{\delta \mathcal{F}}{\delta \epsilon} \tag{3.19}$$

Using eq(3.18) J^μ can be rewritten as

$$J^\mu = T_{C,\nu}^{\mu}\frac{\delta x^\nu}{\delta \epsilon} - \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)}\frac{\delta \mathcal{F}}{\delta \epsilon}. \tag{3.20}$$

To determine the conserved current associated with dilatations we consider the infinitesimal transformation

$$x^{'\mu} = x^\mu + \lambda,\ \mathcal{F}\phi = \left(1 - d\lambda\right)\phi \tag{3.21}$$

Here $d = \triangle$ is the scale dimension of the field ϕ. Using the definition (3.20) we find the conserved current of the scale symmetry

$$\mathcal{D}^\mu = x_\nu T_C^{\mu\nu} - d\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)} \tag{3.22}$$

Using the same procedure we find the current associated with SCT as follows

$$\mathcal{K}^{\mu\nu} = 2x^\nu x_\rho T_C^{\mu\rho} - x^2 T_C^{\mu\nu} - \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi\right)}\left(2dx^\nu + 2ix_\rho S^{\rho\nu}\right) \tag{3.23}$$

The energy momentum-tensor (3.18) is not symmetric. However, it is possible to obtain a symmetric energy- momentum tensor by adding the divergence of an antisymmetric tensor in its first two indices, which does not spoil the conservation of the currents J. This new symmetrized tensor is called Belinfante energy momentum tensor and it is given by

$$T_B^{\mu\nu} = T_C^{\mu\nu} + \partial_\rho B^{\mu\rho\nu} \tag{3.24}$$

Where

$$B^{\mu\rho\nu} = \frac{i}{2}\left(\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi\right)}S^{\rho\nu}\phi + \frac{\partial\mathcal{L}}{\partial\left(\partial_\rho\phi\right)}S^{\mu\nu}\phi + \frac{\partial\mathcal{L}}{\partial\left(\partial_\nu\phi\right)}S^{\rho\mu}\phi\right) \tag{3.25}$$

Since $S^{\mu\nu} = -S^{\nu\mu}$ we can show that the Belinfante tensor is symmetric. With the new tensor $T_B^{\mu\nu}$ the dilatation and SCT currents takes a remarkably simple formulas

$$\mathcal{D}^\mu = x_\nu T_B^{\mu\nu} \tag{3.26}$$

$$\mathcal{K}^{\mu\nu} = x^\nu\mathcal{D}^\mu - x^2 T_B^{\mu\nu} \tag{3.27}$$

which gives the conserved charges

$$D = \int d^3x_\nu T_B^{0\nu} \tag{3.28}$$

$$K^\mu = \int d^3x\left(x^0\mathcal{D}^\mu - x^2 T_B^{0\mu}\right) \tag{3.29}$$

From eq(3.27) we see that if the divergence of $\mathcal{D}^\mu$ vanishes so does the divergence of $\mathcal{K}^{\mu\nu}$ which implies that scale invariance of a theory (expressed by $\partial_\mu\mathcal{D}^\mu = 0$) guarantee its conformal invariance. From both eq(3.26) and eq (3.27) it is evident that the conservation of the dilatations and SCT currents are satisfied only if the energy-momentum tensor has no divergence and is also traceless

$$\partial_\mu T_B^{\mu\nu} = 0, : \eta_{\mu\nu}T_B^{\mu\nu} = 0 \tag{3.30}$$

3.5 Conformal invariance for quantum field theory

Up to now, we have seen conformal transformations and their representations in classical field theory. We saw that conformal invariance is satisfied if the energy momentum tensor is traceless, $T_\nu^\nu = 0$. However, in a quantum theory this condition is not satisfied because renormalization effects introduce a scale μ, called the renormalization scale, which breaks conformal invariance. So, in order to find the condition under which scale invariance is preserved in the quantum theory, we must take into account the variation of gauge coupling under scale transformations. This can be done using the beta function which governs coupling evolution according to the following equation

$$\beta = \frac{\partial g}{\partial\log(\mu)} \tag{3.31}$$

If we perform a scale transformation we get

$$\begin{aligned} x^\nu &\to (1+\epsilon)x^\nu \\ g_i(\mu) &\to g_i\left(\frac{\mu}{1+\epsilon}\right) \approx g_i(\mu-\epsilon\mu) \approx g_i(\mu) - \epsilon\beta_i \end{aligned} \tag{3.32}$$

where β_i is defined as

$$\beta_i = \beta(g_i) = \frac{\partial g_i}{\partial \log(\mu)} \tag{3.33}$$

Now, If we take the classical action $S = \int d^D \mathcal{L}(\Phi, \partial_\nu \Phi)$, which depends on some fields Φ and their derivatives, we can use Noether theorem to write the variation of the action under some transformation in the form

$$\delta S = \int d^D \partial_\nu J^\nu_a \omega^a \tag{3.34}$$

where J^ν is the conserved current associated with some transformation parametrized by ω^a , in our case the scale transformation. According to Noether theorem, in order for the classical theory to be invariant under such transformation, we must have $\delta S = 0$, but the evolution of gauge coupling in the quantum theory induced by the renormalization procedure add a quantum term to equation (3.34)

$$\begin{aligned} \delta S &= \delta S_{class} + \delta S_{quant} \\ &= \int d^D \partial_\nu J^\nu_a \omega^a + \int d^D \frac{\partial \mathcal{L}}{\partial g_i} \delta g_i \\ &= \int d^D \partial_\nu J^\nu_a \omega^a - \int d^D \frac{\partial \mathcal{L}}{\partial g_i} \epsilon \beta_i \end{aligned} \tag{3.35}$$

where the δg_i is replaced form eq(3.32). Setting $\delta S = 0$ we find

$$\partial_\nu J^\nu = \frac{\partial \mathcal{L}}{\partial g_i} \beta_i \tag{3.36}$$

Using the fact that the current for a scale transformation is $J^\nu = D^\nu = T^\nu_\lambda x^\lambda$ we find

$$T^\nu_\nu = \frac{\partial \mathcal{L}}{\partial g_i} \beta_i \tag{3.37}$$

So, scale invariance for a quantum field theory is directly related to the annihilation of the beta function $\beta_i = 0$. It is worth mentioning that conformal invariance in quantum theory is inferred from scale invariance because no example so far have been found of a theory that satisfies scale invariance and at the same time violates conformal invariance.

3.6 Correlation functions

To perform calculations in a quantum field theory, we need to compute correlation functions because the S matrix is not a useful tool in conformal field theory. Due to scale invariance, the concept of a particle being far away is equivalent to a particle being very close to the centre of the event. Conformal invariance imposes a severe restrictions on the form of correlation functions, especially two point function, which we calculate in the following for a quasi-primary scalar field ϕ.

The expression of two point function is the following

$$\left\langle \phi^{j1}(x_1^{'})\phi^{j2}(x_2^{'})\right\rangle = \frac{1}{Z}\int \mathcal{D}[\phi]\phi^{j1}\left(x_1\right)\phi^{j2}\left(x_2\right)e^{-S(\phi)} \tag{3.38}$$

where the partition function Z is defined by

$$Z = \int \mathcal{D}[\phi]e^{-S[\phi]} \tag{3.39}$$

Since the action S and the integration measure $\mathcal{D}[\phi]$ must both be invariant Under CT, we can deduce from (3.38) the transformation rules for two point correlation function

$$\left\langle \phi^{j1}\left(x_1\right)\phi^{j2}\left(x_2\right)\right\rangle = \left|\frac{\partial x'}{\partial x}\right|_{x=x_1}^{d_1/D}\left|\frac{\partial x'}{\partial x}\right|_{x=x_2}^{d_2/D}\left\langle \phi^{j1}(x_1^{'})\phi^{j2}x_2^{'})\right\rangle \tag{3.40}$$

d_i is the scaling dimension of the field ϕ^j and j its spin. We consider here spineless fields for simplicity and we drop the upper-case j. From eq (3.13) we found that the jacobian of a conformal transformation is given by

$$\left|\frac{\partial x'}{\partial x}\right| = \frac{1}{\sqrt{det\ \eta_{\mu\nu}^{'}}} = \Omega(x)^{-D/2} \tag{3.41}$$

In the case of dilatation $\Omega(x)$=λ. then, the transformation rule (3.40) becomes

$$\left\langle \phi\left(x_1\right)\phi\left(x_2\right)\right\rangle = \lambda^{d_1+d_2}\left\langle \phi(\lambda x_1)\phi\lambda x_2)\right\rangle \tag{3.42}$$

Invariance under translations and rotations means that the correlations functions must only depends on the distances $|x_1 - x_2|$. In this case, the two point function takes the general form

$$\left\langle \phi\left(x_1\right)\phi\left(x_2\right)\right\rangle = f\left(|x_1 - x_2|\right) \tag{3.43}$$

Combing the conditions (3.42) and (3.43), the two point function takes the form

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{d_1+d_2}} \tag{3.44}$$

where C_{12} is a normalization factor. The SCT imposes a further restriction on the two point function as follows

$$\frac{C_{12}}{|x_1 - x_2|^{d_1+d_2}} = \frac{C_{12}}{\gamma_1^{d_1}\gamma_2^{d_2}}\frac{(\gamma_1\gamma_2)^{(d_1+d_2)/2}}{|x_1 - x_2|^{d_1+d_2}} \tag{3.45}$$

with $\gamma_i = (1 - bx_i + b^2x_i^2)$. This constraint is satisfied only if $d_1 = d_2$ otherwise, the two point function vanishes. In other word, quasi-primary fields are correlated only if they have the same scaling dimension

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \begin{cases} \dfrac{C_{12}}{|x_1 - x_2|^{2d}}, & \text{if } d_1 = d_2 \\ 0, & \text{if } d_1 \neq d_2 \end{cases} \tag{3.46}$$

A similar analysis can be carried out for fermionic fields ($j = 1/2$) ψ with scaling dimension d_f, vector fields($j = 1$) V^μ with scaling dimension d_V and tensor fields ($j = 2$) $T^{\mu\nu}$ with scaling dimension d_T. Here we just give the final results for the two points functions for each case. We take $x_1 = x$ and $x_2 = 0$

$$\left\langle \phi^{j_1}(x)\phi^{j_2}(0)\right\rangle = C_s\frac{\delta_{j_1j_2}}{(2\pi)^2}\frac{1}{(x^2)^{d_S}} \tag{3.47}$$

$$\left\langle \psi^{j_1}(x)\bar{\psi}^{j_2}(0)\right\rangle = C_f\frac{\delta_{j_1j_2}}{(2\pi)^2}\frac{x_\mu\gamma^\mu}{(x^2)^{d_f+1/2}} \tag{3.48}$$

and

$$\left\langle V_\mu^{j_1}(x)V_\nu^{j_2}(0)\right\rangle = C_v\frac{\delta_{j_1j_2}}{(2\pi)^2}\frac{I_{\mu\nu}}{(x^2)^{d_v}} \tag{3.49}$$

and

$$\left\langle T_{\mu\nu}^{j_1}(x)T_{\rho\sigma}^{j_2}(0)\right\rangle = C_t\frac{\delta_{j_1j_2}}{(2\pi)^2}\frac{I_{\mu\rho}(x)I_{\nu\sigma}(x) - \frac{1}{D}\eta_{\mu\nu}\eta_{\rho\sigma} \pm \mu \leftrightarrow \nu}{(x^2)^{d_t}} \tag{3.50}$$

with $I_{\mu\nu} = \eta_{\mu\nu} - 2x_\mu x_\nu$ which is fixed by demanding invariance under the SCT.

3.7 Unitarity constraints

The unitarity condition imposed on conformal fields allow us to find a lower bound on the scale dimension d for each type of field, scalar, fermionic, vector, or tensor. This condition entails that the normalization constants of the correlation functions (3.47,3.48,3.49,3.50) must be positive , $C_i > 0$ [26], this condition leads to the following constraint

$$d \geq j_1 + j_2 + 2 - \delta_{j_1,j_2} \tag{3.51}$$

where j_1, j_2 are the spins of the corresponding fields. So, for scalar fields ϕ, fermionic fields ψ, vectorial fields V^μ and tensorial fields $T^{\mu\nu}$ we get the following restrictions on their scale dimension

$$\begin{aligned} d_s &\geq 1 \\ d_f &\geq \frac{3}{2} \\ d_V &\geq 3 \\ d_T &\geq 4 \end{aligned} \tag{3.52}$$

In addition to the unitarity constraint, if the vector and tensor fields obey the conservation requirements $\partial^\mu V_\mu = 0$ and $\partial^\mu T_{\mu\nu} = 0$, their scale dimensions d_V and d_T take the canonical values $d = 3$ and $d = 4$ respectively.

3.8 Ward Identities

Ward identities allow us to restate the conservations laws, expressed by Noethers theorem, in the form of operator equations. The general identity for a conserved current J is the following

$$\frac{\partial}{\partial x^\mu} \langle J^\mu(x)\phi(x_1)\ldots\phi(x_n)\rangle = -i\sum_i \delta(x-x_i)\langle\phi(x_1)\ldots G\phi(x_i)\ldots\phi(x_n)\rangle \tag{3.53}$$

where G is the generator of the conformal transformation associated with J. We use the notation X to denote the product of n operators

$$X = \phi(x_1)\ldots\phi(x_n)$$

The ward identity associated with translation invariance can be written by identifying the current J with the modified energy momentum tensor $T_B^{\mu\nu}$ and the generator G with the momentum operator $P^\mu = \dfrac{\partial}{\partial x^\mu}$ as follows

$$\frac{\partial}{\partial x^\mu}\langle T^\mu_\nu(x)X\rangle = -i\sum_i \delta(x-x_i)\left\langle\frac{\partial X}{\partial x^\nu}\right\rangle \tag{3.54}$$

Now, we use eq(3.12), which defines the generators of the conformal group and the set of eq(3.42) and eq(3.27) to deduce ward identities for rotations(Lorentz transformations) and dilatations. For rotations the modified angular momentum currents is

$$M^{\mu\nu,\rho} = (T^{\mu\nu}x^\rho - T^{\mu\rho}x^\nu)$$

and the generator of Lorentz transformation is given in eq(3.12). Therefore, the ward identity is

$$\partial_\mu \langle T^{\mu\nu}x^\rho - T^{\mu\rho}x^\nu \; X\rangle = i\sum_i \delta(x - x_i)\,(x_i^\nu \partial_i^\rho - x_i^\rho \partial_i^\nu - iS_i^{\nu\rho})\,\langle X\rangle$$

using the first ward identity (3.54), we can rewrite the previews equation as

$$\langle T^{\rho\nu} - T^{\nu\rho} \; X\rangle = \sum_i \delta(x - x_i)S_i^{\nu\rho} \tag{3.56}$$

where $S_i^{\nu\rho}$ is the spin matrix for the i-th field of X. Applying the same procedure using the dilatation currents $D^\mu = T^\mu_\nu x^\nu$ and the associated generator

$$D = -i\,(x^\mu\partial_\mu + d)$$

the ward identity for dilatation is

$$\partial_\mu \langle T^\mu_\nu x^\nu \; X\rangle = -\sum_i \delta(x - x_i)\,(x_i^\nu \partial_{i,\nu} + d_i)\,\langle X\rangle \tag{3.57}$$

Using (3.54) again, we find

$$\left\langle T^\mu_\mu X\right\rangle = -\sum_i \delta(x - x_i)d_i\,\langle X\rangle \tag{3.58}$$

where d_i is the scaling dimension of the i-th field of the product X. Eqs (3.54), (3.56), and (3.58) are the Ward identities corresponding to the conformal group.

3.9 Effective field theories

In physics we need to describe physical phenomenons that take place at different scales. Since a fundamental theory of particle physics is not yet known, we must contend with describing interactions in a particular scale with fields relevant to that scale. The theoretical tool to achieve this goal is effective field theory(EFT), which is a field theory that describes physics below some energy scale Λ, as opposed to a fundamental theory which is valid in all energy scales. In fact, every field theory known to date is an effective field theory, including the SM. The fact that we know that our theory is valid up to a scale free us from the requirement of renormalizibility which represents the biggest challenge in constructing a fundamental theory. One only need to build an EFT with finite number of parameters that describe physics up to effects suppressed by $\left(\frac{E}{\Lambda}\right)$, where E is the energy of the particles in the effective theory. The approach used in EFT is the Wilsonian approach [27, 28] which consists in taking the lagrangian of a fundamental theory splitting the fields into light modes and heavy modes, and then integrating out the heavy fields above some cutt-off Λ. However, In practical calculations, it is much easier to work with dimensional regularization in the $\overline{MS}$ scheme instead of the cuttuff Λ . To be more concrete let us consider a field theory with a characteristic energy scale Λ. We are only interested in physics at low energy $E \ll \Lambda$. The dynamic of the full theory can be described by correlation function defined by the following path integral

$$\langle 0|\, T\; \phi(x_1)\ldots\phi(x_n)\; |0\rangle = \frac{1}{Z}\int \mathcal{D}\phi e^{iS(\phi)}\phi(x_1)\ldots\phi(x_n) \tag{3.59}$$

where $\mathcal{D}\phi = \prod_{x_i} d\phi(x_i)$ and $Z = \int \mathcal{D}\phi e^{iS(\phi)}$. To obtain the low energy effective theory, we split the field ϕ into

$$\phi = \phi_l + \phi_h \tag{3.60}$$

where ϕ_h contain all Fourier modes with $\omega > \Lambda$ and ϕ_l contains the low energy modes $\omega < \Lambda$. Since we are only interested in low energy physics, we keep the light modes and integrate out the heavy modes. The resulting correlation function is

$$\langle 0|\, T\; \phi_l(x_1)\ldots\phi_l(x_n)\; |0\rangle = \int \mathcal{D}\phi_l e^{iS_\Lambda(\phi_l)}\phi_l(x_1)\ldots\phi_l(x_n) \tag{3.61}$$

where $S_\Lambda(\phi_l)$ is defined by

$$S_\Lambda = \int \mathcal{D}\phi_h e^{iS[\phi_l,\phi_h]} \tag{3.62}$$

$S[\phi_l, \phi_h]$ being the full action from eq(3.59). S_Λ is called the wilsonian effective action. Because we have eliminated the high energy modes we end up with a non-local action according to uncertainty principle ($\Delta x \sim \frac{1}{\Lambda} \neq 0$). We now expand the non-local action as a series of local operators. This is allowed because $E \ll \Lambda$. The result has the form

$$S_\Lambda = \int dx L_\Lambda^{eff}(x) \tag{3.63}$$

with

$$L_\Lambda^{eff} = \sum g_i O_i(x) \tag{3.64}$$

L_Λ^{eff} is called the effective lagrangian and g_i are referred to as Wilson coefficients. If we perform a dimensional analysis on the lagrangian (3.64), we find that the coefficient g_i can be written as

$$g_i = c_i \frac{1}{\Lambda^{d_i - D}} \tag{3.65}$$

where c_i is dimensionless coupling constant and d_i is the scaling dimension of the operator O_i. Since g_i, hence c_i, is obtained by integrating out the heavy degree of freedom, all the information about the high energy scale M (the scale of the fundamental theory) is encoded in c_i. We can classify the operators O_i into three categories according to their dimensions d_i

- Relevant if ($d_i < D$)
- Marginal if ($d_i = D$)
- Irrelevant if ($d_i > D$)

The operator O_i with $d_i > D$ are called irrelevant because they are suppressed by power of $\left(\frac{E}{\Lambda}\right)$ and become small at low energies. Operators with $d_i = D$ are called marginal or renormalizable and they can teal toward relevancy or irrelevancy because quantum effects could modify their scaling behavior on either side. Operators with $d_i < D$ are relevant because their effect becomes more important as the energy decreases.

Chapter 4

Unparticle model

4.1 Introduction

Unparticle model was proposed by Howard Georgi in 2007 [3] as a theoretical curiosity. He suggested the existence of scale invariant hidden sector interacting with SM particles at high energies with a connector sector, a messenger that carries both the SM quantum numbers and the hidden sector quantum numbers. The use of a mediator is a way of simplifying the complicated and nonlinear interactions at high energies. Georgi has chosen to model these scale invariants fields with the so called Banks-Zaks fields (BZ) proposed in 1984 by the said authors. Banks-Zaks fields [29] are massless fermions which belong to the fundamental representation of the $SU(N)$ group, whose dynamics depends on the numbers of flavours N_F in the fundamental representation. These fields have non-trivial fixed point generated by running the renormalized coupling using renormalization group flow. In his paper, Georgi used these fields to construct an interacting theory between the SM and the hidden sector which flows into a scale invariant theory at low energy. In what follow, we describe briefly the general principles beyond the unparticle model.

Georgi model is based on two interacting sectors, a visible sector (SM fields) and a hidden sector (BZ fields) which communicate with each other through the exchange of very heavy field, of mass M. Using effective field theory as a tool, we can integrate out the mediator fields and we end up with an effective Lagrangian describing the interactions as follows

$$L \propto c_0 M^{d_v+d_h-D} O_v O_h + c_1 M^{2d_v-D} O_v O_v + c_2 M^{2d_v+2-D} O_v \partial^2 O_v + \dots \quad (4.1)$$

where O_v are the operators representing the SM fields and O_h represent the BZ fields. The coefficients c_i are dimensionless coupling constants. The first term in this Lagrangian represents the interaction between BZ fields and the SM fields. The second term is a contact interaction between the SM fields. The other terms are non-renormalizable interactions suppressed by power of M. So, the term relevant to our model is the first one, which can be expressed in the generic form

$$\frac{1}{M^c} O_{SM} O_{BZ} \tag{4.2}$$

If we decrease the energy further, according to BZ [29], a scale invariant sector emerges at a scale Λ. Below this scale dimensional transmutation occurs. That is the dimensionless coupling c_i transmute into dimensionful coupling and the BZ operators match onto unparticle operators. The interaction (4.2) becomes

$$C_{\mathcal{U}} \frac{\Lambda^{d_{BZ}-d_{\mathcal{U}}}}{M} O_{SM} O_{\mathcal{U}} \tag{4.3}$$

where $d_{\mathcal{U}}$ is the scaling dimension of the unparticle operator $O_{\mathcal{U}}$.

Describing unparticle physics using the term (4.3) has multiple advantages:

1. In the low energy theory, below the scale Λ, the BZ fields decouple from the SM fields which help us keep the IR theory (unparticles) scale invariant, otherwise the situation would have been a lot more complicated.

2. If we make the mass of the mediator field M large enough, the coupling between the SM and unparticles becomes very weak at low energy, which explain the absence of detection of these novel fields in currents colliders.

Because of scale invariance, unparticles do not obey the usual dispersion relation $\omega^2 = c^2k^2 + m^2c^4/\hbar$. Unparticles do not have a definite mass. They are either massless fields or they have a continuous mass distribution. This is because the presence of a mass term in the Lagrangian destroys scale invariance. If we take the dilatation operator D, it can be shown that

$$e^{i\alpha D} P^2 e^{-i\alpha D} = e^{2\alpha} P^2 \tag{4.4}$$

If we apply this equation on the vacuum state, it shows that scale invariance cannot exist in a massive theory. Thus, conformal symmetry must be spontaneously broken. To prove this concept let us consider discrete massive state with $p^2 = m^2$, using the conformal algebra of eq(3.5), we find

$$[\,P^2,\, D\,] = P^\mu [\,P_\mu,\, D\,] + [\,P^\mu,\, D\,] P_\mu = 2iP^2 \tag{4.5}$$

Then,

$$\langle p|\,[\,P^2,\,D\,]\,|p\rangle = 2i\,\langle p|\,P^2\,|p\rangle = 2im^2, \tag{4.6}$$

in the other hand we have

$$\langle p|\,[\,P^2,\,D\,]\,|p\rangle = \langle p|\,m^2D - Dm^2\,|p\rangle = 0 \tag{4.7}$$

which means $m^2 = 0$. Hence, all massive states break conformal symmetry. A conformal invariant theory should only contains massless states or a continuous mass spectrum.

4.2 Phase space

In order to compute the probability distribution involving unparticle states, we have to find the phase space of unparticles. For this reason, we start with the Whitman correlation function defined as

$$W(x) = \langle 0|\,O_{\mathcal{U}}(x)O^{\dagger}_{\mathcal{U}}(0)\,|0\rangle \tag{4.8}$$

If we insert the complete set of intermediate unparticles momentum states

$$1 = \int d\lambda\,|\lambda\rangle\,\langle\lambda| \tag{4.9}$$

and we use the properties of the translation operator P, we find

$$\begin{aligned}\langle 0|\,O_{\mathcal{U}}(x)O^{\dagger}_{\mathcal{U}}(0)\,|0\rangle &= \langle 0|\,e^{iPx}O_{\mathcal{U}}(0)e^{-iPx}O^{\dagger}_{\mathcal{U}}(0)\,|0\rangle \\ &= \int d\lambda \int d\lambda'\,\langle 0|\,O_{\mathcal{U}}(0)\,|\lambda'\rangle\,\langle\lambda'|\,e^{-iPx}\,|\lambda\rangle\,\langle\lambda|\,O^{\dagger}_{\mathcal{U}}(0)\,|0\rangle \\ &= \int d\lambda e^{-ip_{\lambda}.x}|\langle 0|\,O_{\mathcal{U}}(0)\,|\lambda\rangle|^2 \end{aligned} \tag{4.10}$$

inserting the unity relation $\int \frac{d^4p}{(2\pi)^4}\delta^4\left(p - p_{\lambda}\right) = 1$, we can rewrite the Whitman function (4.10) as

$$\langle 0|\,O_{\mathcal{U}}(x)O^{\dagger}_{\mathcal{U}}(0)\,|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip.x}\rho_{\mathcal{U}}(p^2) \tag{4.11}$$

where $\rho_{\mathcal{U}}(p^2)$ is a Lorentz invariant spectral density function defined by

$$\begin{aligned}\rho_U(p^2) &= (2\pi)^4 \int d\lambda\delta^4\left(p - p_{\lambda}\right)|\langle 0|\,O_{\mathcal{U}}(0)\,|\lambda\rangle|^2 \\ &= (2\pi)^4\,\theta(p^0)\theta(p^2)\widetilde{\rho}_{\mathcal{U}}(p^2)\end{aligned} \tag{4.12}$$

the heavyside step function $\theta(p^0)$ ensures that the spectral function contains only positive energies and $\theta(p^2)$ ensure that unparticles are not tackyonic. If we substitute the second equation from (4.12) in eq(4.11) we get the following

$$\langle 0| O_{\mathcal{U}}(x)O_{\mathcal{U}}(0) |0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip.x}\theta(p^0)\theta(p^2)\widetilde{\rho}_{\mathcal{U}}(p^2) \tag{4.13}$$

In the other hand, from eq(3.46) we see that conformal symmetry imposes a particular form on the two point correlation function. For scalar field it is given by

$$\langle 0| O_{\mathcal{U}}(x)O_{\mathcal{U}}(0) |0\rangle = C_s \frac{1}{(2\pi)^2}\frac{1}{(x^2)^d} \tag{4.14}$$

In momentum space this expression translates to

$$\langle 0| O_{\mathcal{U}}(x)O_{\mathcal{U}}(0) |0\rangle = C_s \frac{\Gamma(2-d)}{4^{d-1}\Gamma(d)} \int \frac{d^4p}{(2\pi)^4} e^{ip.x}(p^2)^{d-2} \tag{4.15}$$

Now, comparing eq(4.13) and eq(4.15), we find

$$\widetilde{\rho}_{\mathcal{U}}(p^2) = A(d)(p^2)^{d-2} \tag{4.16}$$

where $A(d)$ is a normalization factor to be determined. Using eq(4.12), the spectral density for unparticle states is

$$\rho_{\mathcal{U}}(p^2) = A(d)\theta(p^0)\theta(p^2)(p^2)^{d-2} \tag{4.17}$$

this expression allow us to define the phase space of an unparticle state with momentum p as follows

$$d\Phi_{\mathcal{U}}(p) = \rho_{\mathcal{U}}(p^2)\frac{d^4p}{(2\pi)^4} = A(d)\theta(p^0)\theta(p^2)(p^2)^{d-2}\frac{d^4p}{(2\pi)^4} \tag{4.18}$$

We see that this expression is similar to the phase space of n massless scalar particles:

$$\begin{aligned} d\Phi_n(p) &= \prod_{i=1}^{n} \frac{d^4p_i}{(2\pi)^3}\delta\left(p_i^2\right)\theta\left(p_i^0\right)(2\pi)^2\delta^4\left(p-\sum_{i=1}^{n}p_i\right)\frac{d^4p}{(2\pi)^4} \\ &= A(n)\theta(p^0)\theta(p^2)(p^2)^{n-2}\frac{d^4p}{(2\pi)^4} \end{aligned} \tag{4.19}$$

where

$$A(n) = \frac{16\pi^2\sqrt{\pi}}{(2\pi)^{2n}}\frac{\Gamma\left(n+1/2\right)}{\Gamma(n-1)\Gamma(2n)} \tag{4.20}$$

So, if we take the limit $n \to d$ and $A(n) \to A(d)$, the normalization factor (4.20) takes the form

$$A(d) = \frac{16\pi^2\sqrt{\pi}}{(2\pi)^{2d}} \frac{\Gamma\left(d + 1/2\right)}{\Gamma(d-1)\Gamma(2d)} \tag{4.21}$$

d, the scale dimension of unparticles, can take fractional values which is a peculiar feature of unparticle physics. The phase space of an unparticle state resembles that of a fractional number of massless particles. Or as Georgi put it

"Unparticle stuff with scale dimension d look like a non-integral number of invisible particles"

If we take the limit $d \to 1$, we can recover the phase space of an ordinary scalar particle as follows

$$\lim_{d\to 1} A(d) = 2\pi, \ \lim_{d\to 1}(p^2)^{d-2} = \frac{1}{p^2} \tag{4.22}$$

Using the propriety $\lim_{\epsilon\to 0} \frac{\epsilon\theta(x)}{x^{1-\epsilon}} = \delta(x)$, we find

$$\lim_{d\to 1} A(d)\theta(p^0)\theta(p^2)(p^2)^{d-2} = 2\pi\theta(p^0)\delta(p^2) \tag{4.23}$$

So,

$$d\Phi_{d\to 1}(p^2) = 2\pi\theta(p^0)\delta(p^2)\frac{d^4p}{(2\pi)^4} \tag{4.24}$$

which is indeed the phase space of a massless scalar particle.

4.3 Propagators

In this section we use the spectral density, constrained by conformal symmetry, and the Kallen-Lemman spectral representation of propagators [30, 31] to find the propagators in momentum space for unparticles fields with different spins. We start by considering a scalar unparticle $\mathcal{U}_s$. We can define the propagator in configuration space as follows

$$\Delta_{\mathcal{U}_s}(x-y) = \int \frac{d^Dp}{(2\pi)^D} e^{ip(x-y)} \widetilde{\Delta}_{\mathcal{U}_s}(p^2) \tag{4.25}$$

where $\widetilde{\Delta}_{\mathcal{U}_s}$ is the propagator of scalar unparticle in momentum space. We start our derivation by the definition of propagator in configuration space

$$\Delta_{\mathcal{U}_s}(x-y) = \langle 0| T\mathcal{U}_s(x)\mathcal{U}_s(y) |0\rangle \tag{4.26}$$

where T denote the time ordered product of the $\mathcal{U}_s s$ operators. Inserting the completeness relation

$$\int d\lambda \left|\lambda\right\rangle \left\langle\lambda\right| = 1 \tag{4.27}$$

between the operators $\mathcal{U}_s(x)$ and $\mathcal{U}_s(y)$, we find

$$\Delta^+_{\mathcal{U}_s}(x-y) = \int d\lambda \left\langle 0\right| \mathcal{U}_s(x) \left|\lambda\right\rangle \left\langle\lambda\right| \mathcal{U}_s(y) \left|0\right\rangle \tag{4.28}$$

here the $+$ in Δ indicates that $x^0 > y^0$, so we dropped the time ordered product T. Now, using the transformation property

$$\mathcal{U}_s(x) = e^{-iP\cdot x}\mathcal{U}_s(0)e^{iP\cdot x} \tag{4.29}$$

where P is the momentum operator which satisfy $P\left|\lambda\right\rangle = p_\lambda \left|\lambda\right\rangle$, eq(4.28) becomes

$$\begin{aligned}\Delta^+_{\mathcal{U}_s}(x-y) &= \int d\lambda \left\langle 0\right| e^{-iP\cdot x}\mathcal{U}_s(0)e^{iP\cdot x} \left|\lambda\right\rangle \left\langle\lambda\right| e^{-iP\cdot y}\mathcal{U}_s(0)e^{iP\cdot y} \left|0\right\rangle \\ &= \int d\lambda e^{ip_\lambda\cdot(x-y)} |\left\langle 0\right| \mathcal{U}_s(0) \left|\lambda\right\rangle|^2 \end{aligned} \tag{4.30}$$

Now inserting the unity relation

$$\int \delta(p-p_\lambda)\frac{dp^D}{(2\pi)^D} = 1$$

in eq(4.30) we get the following formula

$$\Delta^+_{\mathcal{U}_s}(x-y) = \int \frac{dp^D}{(2\pi)^D} \int d\lambda e^{ip\cdot(x-y)} |\left\langle 0\right| \mathcal{U}_s(0) \left|\lambda\right\rangle|^2 \delta(p-p_\lambda) \tag{4.31}$$

Next, we define the spectral density

$$\rho(p^2) = \int |\left\langle 0\right| \mathcal{U}_s(0) \left|\lambda\right\rangle|^2 \delta(p-p_\lambda) d\lambda$$

which is the same spectral density defined in the previews section (eq(4.12)). so, we can write

$$\rho_{\mathcal{U}}(p^2) = \theta(p^2)\theta(p^0)\widetilde{\rho}_{\mathcal{U}}(p^2) \tag{4.32}$$

with

$$\widetilde{\rho}_{\mathcal{U}}(p^2) = A(d)(p^2)^{d_s-2} \tag{4.33}$$

Now, we come back to the time ordered green function $\Delta_{\mathcal{U}_s}(x-y)$. We can write it as

$$\langle 0|\, T\mathcal{U}_s(x)\mathcal{U}_s(y)\,|0\rangle = \theta(x^0-y^0)\,\langle 0|\,\mathcal{U}_s(x)\mathcal{U}_s(y)\,|0\rangle + \theta(y^0-x^0)\,\langle 0|\,\mathcal{U}_s(y)\mathcal{U}_s(x)\,|0\rangle \tag{4.34}$$

where $\langle 0|\,\mathcal{U}_s(y)\mathcal{U}_s(x)\,|0\rangle = \Delta^+_{\mathcal{U}_s}(y-x)$ is the propagator for $y^0 > x^0$. $\Delta^+_{\mathcal{U}_s}(x-y)$ and $\Delta^+_{\mathcal{U}_s}(y-x)$ can be rewritten in the following form

$$\begin{aligned}\Delta^+_{\mathcal{U}_s}(x-y) &= \int_0^\infty dM^2\rho(M^2)\int \frac{dp^D}{(2\pi)^D} e^{ip\cdot(x-y)}\delta(p^2+M^2)\\ &= \int_0^\infty dM^2\rho(M^2)\Delta^+(x-y;M^2)\end{aligned} \tag{4.35}$$

and

$$\begin{aligned}\Delta^+_{\mathcal{U}_s}(y-x) &= \int_0^\infty dM^2\rho(M^2)\int \frac{dp^D}{(2\pi)^D} e^{ip\cdot(y-x)}\delta(p^2+M^2)\\ &= \int_0^\infty dM^2\rho(M^2)\Delta^+(y-x;M^2)\end{aligned} \tag{4.36}$$

Using Eqs(4.35,4.36), the time ordered propagator function is

$$\langle 0|\, T\mathcal{U}_s(x)\mathcal{U}_s(y)\,|0\rangle = -i\int_0^\infty dM^2\rho(M^2)\Delta_F(x-y;M^2) \tag{4.37}$$

where $\Delta_F(x-y;M^2)$ is the Feynman propagator for a spinless particle of mass M:

$$-i\Delta_F(x-y;M^2) = \theta(x^0-y^0)\Delta^+(x-y;M^2) - \theta(y^0-x^0)\Delta^+(y-x;M^2) \tag{4.38}$$

the scalar unparticle propagator in momentum space is

$$\widetilde{\Delta}_{\mathcal{U}_s}(p^2) = \int d^Dx\,\exp(-ip\cdot(x-y))\,\langle 0|\, T\mathcal{U}_s(x)\mathcal{U}_s(y)\,|0\rangle \tag{4.39}$$

Knowing that

$$\int d^Dx\,\exp(-ip\cdot(x-y)))\Delta_F(x-y;M^2) = \frac{1}{p^2+M^2-i\epsilon} \tag{4.40}$$

this yields the spectral representation of unparticle propagator:

$$\widetilde{\Delta}(p^2) = \int_0^\infty dM^2 \frac{\rho(M^2)}{p^2+M^2-i\epsilon} = \int_0^\infty dM^2 \frac{A(d)(M^2)^{d_s-\frac{D}{2}}}{p^2+M^2-i\epsilon} \tag{4.41}$$

performing the integration we get finally

$$\widetilde{\Delta}_{\mathcal{U}_s}(p^2) = \frac{A(d_{\mathcal{U}_s})}{2sin(\pi d_{\mathcal{U}_s})} \frac{-i}{(-p^2 - i\epsilon)^{D/2-d_{\mathcal{U}_s}}} \tag{4.42}$$

in 4 spacetime dimension $D = 4$, this expression becomes

$$\widetilde{\Delta}_{\mathcal{U}_s}(p^2) = \frac{A(d_{\mathcal{U}_s})}{2sin(\pi d_{\mathcal{U}_s})} \frac{-i}{(-p^2 - i\epsilon)^{2-d_{\mathcal{U}_s}}} \tag{4.43}$$

the factor $(-p^2 - i\epsilon)^{2-d_{\mathcal{U}_s}}$ can be expanded in the following way

$$\begin{aligned} \left(-p^2 - i\epsilon\right)^{2-d_{\mathcal{U}_s}} &= \left(p^2\right)^{2-d_{\mathcal{U}_s}} \left((-1 - i\epsilon)^{2-d_{\mathcal{U}_s}} - (-1 + i\epsilon)^{2-d_{\mathcal{U}_s}}\right) \\ &\approx \left(p^2\right)^{2-d_{\mathcal{U}_s}} \left(e^{-i(d_{\mathcal{U}_s}-2)\pi} - e^{-i(d_{\mathcal{U}_s}-2)\pi}\right) \\ &= \left(p^2\right)^{2-d_{\mathcal{U}_s}} \left(-2i \sin(d_{\mathcal{U}_s}\pi)\right) \end{aligned} \tag{4.44}$$

So eq(4.43) simplifies to

$$\widetilde{\Delta}_{\mathcal{U}_s}(p^2) = A(d_{\mathcal{U}_s}) \left(p^2\right)^{d_{\mathcal{U}_s}-2} \tag{4.45}$$

in the limit $d_{\mathcal{U}_s} \to 1$ we recover the familiar propagator of an ordinary scalar field

$$\lim_{d_{\mathcal{U}_s}\to 1} \widetilde{\Delta}_{\mathcal{U}_s}(p^2) = \frac{1}{p^2} \tag{4.46}$$

4.3.1 Fermionic unparticle

Proceeding in the same manner as in the scalar case, we first consider the green function of fermionic unparticles

$$\begin{aligned} \langle 0|\, T\mathcal{U}_f(x)\mathcal{U}_f(0)\,|0\rangle &= \int d\lambda\, \langle 0|\, T\mathcal{U}_f(x)\,|\lambda\rangle\, \langle\lambda|\, T\mathcal{U}_f^{\dagger}(0)\,|0\rangle \\ &= \int d\lambda e^{ip_\lambda\cdot x} |\langle 0|\, \mathcal{U}_f(0)\,|\lambda\rangle|^2 \\ &= \int d^4p e^{ip\cdot x} \int d\lambda \delta(p - p_\lambda) |\langle 0|\, \mathcal{U}_f(0)\,|\lambda\rangle|^2 \\ &= \int d^4p e^{ip\cdot x} \theta(p^2)\theta(p^0)\widetilde{\rho}(p^2) \end{aligned} \tag{4.47}$$

where $\widetilde{\rho}(p^2)$ is the spectral density corresponding to unfermions in the final states.

In the other hand, we can use conformal symmetry to determine the two point function in configuration space of fermionic fields, the result is the following

$$\langle 0| T\mathcal{U}_f(x)\mathcal{U}_f(0)|0\rangle = \frac{C_f}{(2\pi)^2}\frac{\not{x}}{(-x^2+i\epsilon)^{d_{\mathcal{U}_f}}} \tag{4.48}$$

this expression can be transformed to momentum space via Fourier transformation as follows

$$\langle 0| T\mathcal{U}_f(x)\mathcal{U}_f(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} A(d_{\mathcal{U}_f}-1/2)\not{p}\left(p^2\right)^{d_{\mathcal{U}_f}-5/2} \tag{4.49}$$

comparing equation (4.49) to eq(4.47) we can deduce the expression for the spectral density $\rho(p^2)$

$$\begin{aligned}\rho(p^2) &= A(d_{\mathcal{U}_f}-1/2)\theta(p^2)\theta(p^0)\not{p}\left(p^2\right)^{d_{\mathcal{U}_f}-5/2}\\ &= \theta(p^2)\theta(p^0)\widetilde{\rho}(p^2)\end{aligned} \tag{4.50}$$

Now, using the Källén–Lehmann spectral representation (4.41), we compute the propagator in momentum space

$$\Delta_{\mathcal{U}_f}(p^2) = \frac{1}{2\pi}\int_0^\infty dM^2 \frac{\rho(M^2)}{p^2-M^2+i\epsilon} \tag{4.51}$$

$$= \frac{A(d_{\mathcal{U}_f}-1/2)}{2\pi}\int_0^\infty dM^2 \frac{\left(M^2\right)^{d_{\mathcal{U}_f}-5/2}}{p^2-M^2+i\epsilon}\not{p} \tag{4.52}$$

$$= \frac{A(d_{\mathcal{U}_f}-1/2)}{2\cos\left(\pi d_{\mathcal{U}_f}\right)}\frac{i\not{p}}{(-p^2-i\epsilon)^{5/2-d_{\mathcal{U}_f}}} \tag{4.53}$$

Using the same method described in eq(4.44), this expression simplifies to

$$\Delta_{\mathcal{U}_f}(p^2) = -A(d_{\mathcal{U}_f}-1/2)\not{p}(p^2)^{5/2-d_{\mathcal{U}_f}} \tag{4.54}$$

4.3.2 Vector unparticles

For vector unparticles the Whittman function is defined as

$$\begin{aligned}\langle 0| T\mathcal{U}_v^\mu(x)\mathcal{U}_v^\nu(0)|0\rangle &= \int d\lambda \langle 0|\mathcal{U}_v^\mu(x)|\lambda\rangle\langle\lambda|\mathcal{U}_v^\nu(0)|0\rangle\\ &= \int d\lambda e^{-ip_\lambda\cdot x}\langle 0|\mathcal{U}_v^\mu(x)|\lambda\rangle\langle\lambda|\mathcal{U}_v^\nu(0)|0\rangle\\ &= \int d^4p e^{-ip\cdot x} A(d_{\mathcal{U}_v})\theta(p^2)\theta(p^0)\widetilde{\rho}(p^2)V^{\mu\nu}\end{aligned} \tag{4.55}$$

where the spectral density $\widetilde{\rho}(p^2)$ is defined as before eq(4.33). In the other hand if we Fourier transform the two point function (3.49) for vector conformal fields into momentum space we find

$$\begin{aligned}\langle 0|\, T\mathcal{U}_v^\mu(x)\mathcal{U}_v^\nu(0)\,|0\rangle &= \frac{C_v}{(2\pi)^2}\frac{1}{(-x^2+i\epsilon)}\left(\eta^{\mu\nu}-\frac{2x^\mu x^\nu}{x^2}\right)\\ &= \int \frac{d^4p}{(2\pi)^4}e^{ip\cdot x}A(d_{\mathcal{U}_v})(p^2)^{d_{\mathcal{U}_v}-2}\left(\eta^{\mu\nu}-\frac{2(d_{\mathcal{U}_v}-2)}{d_{\mathcal{U}_v}-1}\frac{p^\mu p^\nu}{p^2}\right)\end{aligned} \tag{4.56}$$

Comparing eq(4.55) to (4.56) we find

$$\rho(p^2) = A(d_{\mathcal{U}_v})\theta(p^2)\theta(p^0)(p^2)^{d_{\mathcal{U}_v}-2} \tag{4.57}$$

and

$$V^{\mu\nu} = \eta^{\mu\nu} - \frac{2(d_{\mathcal{U}_v}-2)}{d_{\mathcal{U}_v}-1}\frac{p^\mu p^\nu}{p^2} \tag{4.58}$$

then, using the spectral representation of the propagator (4.41), we have

$$\begin{aligned}\Delta_{\mathcal{U}_v}(p^2) &= \frac{1}{2\pi}\int_0^\infty dM^2\frac{\rho(M^2)}{p^2-M^2+i\epsilon}V^{\mu\nu}\\ &= \frac{1}{2\pi}\int_0^\infty dM^2\frac{A(d_{\mathcal{U}_v})(M^2)^{d_{\mathcal{U}_v}-2}}{p^2-M^2+i\epsilon}V^{\mu\nu}\\ &= \frac{A(d_{\mathcal{U}_v})}{2\sin(\pi d_{\mathcal{U}_v})}\frac{i\left(\eta^{\mu\nu}-\frac{2(d_{\mathcal{U}_v}-2)}{d_{\mathcal{U}_v}-1}\frac{p^\mu p^\nu}{p^2}\right)}{(-p^2-i\epsilon)^{2-d_{\mathcal{U}_v}}}\end{aligned} \tag{4.59}$$

4.3.3 Tensor unparticles

Due to conformal invariance tensor unparticles are restricted to be either antisymmetric or symmetric and traceless. Pursuing a similar procedure as outlined above, we can construct the propagator for tensor unparticles.

We start by considering the case of antisymmetric unparticle tensor ($\mathcal{U}^{\mu\nu} = -\mathcal{U}^{\nu\mu}$). The two point function is given by

$$\begin{aligned}\langle 0|\, T\mathcal{U}_t^{\mu\nu}(x)\mathcal{U}_t^{\rho\sigma}(0)\,|0\rangle &= \int d\lambda\, \langle 0|\, \mathcal{U}_t^{\mu\nu}(x)\,|\lambda\rangle\, \langle\lambda|\, \mathcal{U}_t^{\rho\sigma}(0)\,|0\rangle\\ &= \int d\lambda e^{-ip_\lambda\cdot x}\, \langle 0|\, \mathcal{U}_t^{\mu\nu}(x)\,|\lambda\rangle\, \langle\lambda|\, \mathcal{U}_t^{\rho\sigma}(0)\,|0\rangle\\ &= \int d^4p e^{-ip\cdot x}A(d_{\mathcal{U}_t})\theta(p^2)\theta(p^0)\widetilde{\rho}(p^2)T^{\mu\nu\rho\sigma}\end{aligned} \tag{4.60}$$

where $\widetilde{\rho}(p^2)$ is the usual spectral density and $T^{\mu\nu\rho\sigma}$ is a function of p that encode the tensorial structure of the propagator (4.60). Now, going back to eq(3.50) we see that conformal symmetry restrict the form of two point function of tensorial fields to be

$$\langle 0| T\mathcal{U}_t^{\mu\nu}(x)\mathcal{U}_t^{\rho\sigma}(0)|0\rangle = C_t\frac{1}{(2\pi)^2}\frac{1}{(x^2)^{d_{\mathcal{U}_t}}}\left([\mathcal{I}^{\mu\rho}(x)\mathcal{I}^{\nu\sigma}(x)+\mu\leftrightarrow\nu]-\frac{1}{2}\eta^{\mu\nu}\eta^{\rho\sigma}\right) \tag{4.61}$$

where $\mathcal{I}^{\mu\rho}(x) = \eta^{\mu\nu} - 2\dfrac{x^\mu x^\nu}{x^2}$. Performing a Fourier transformation to momentum space we get

$$\langle 0| T\mathcal{U}_t^{\mu\nu}(x)\mathcal{U}_t^{\rho\sigma}(0)|0\rangle = \int\frac{d^4p}{(2\pi)^4}e^{ip\cdot x}A(d_{\mathcal{U}_t})(p^2)^{d_{\mathcal{U}_t}-2}T^{\mu\nu\rho\sigma}(p) \tag{4.62}$$

here the tensor structure is encoded in $T^{\mu\nu\rho\sigma}$ as

$$\begin{aligned}T^{\mu\nu\rho\sigma} &= d_{\mathcal{U}_t}(d_{\mathcal{U}_t}-1)(\eta^{\mu\rho}\eta^{\nu\sigma}+\mu\leftrightarrow\nu)+\left(2-\frac{d_{\mathcal{U}_t}}{2}(d_{\mathcal{U}_t}+1)\right)\eta^{\mu\nu}\eta^{\rho\sigma}\\ &-2(d_{\mathcal{U}_t}-1)(d_{\mathcal{U}_t}-2)\left(\eta^{\mu\rho}\frac{p^\nu p^\sigma}{p^2}+\eta^{\mu\sigma}\frac{p^\nu p^\rho}{p^2}+\mu\leftrightarrow\nu\right)\\ &+4(d_{\mathcal{U}_t}-2)\left(\eta^{\mu\nu}\frac{p^\rho p^\sigma}{p^2}+\eta^{\rho\sigma}\frac{p^\mu p^\nu}{p^2}\right)+8(d_{\mathcal{U}_t}-2)(d_{\mathcal{U}_t}-3)\frac{p^\mu p^\nu p^\rho p^\sigma}{(p^2)^2}\end{aligned} \tag{4.63}$$

Comparing eq(4.62) to eq(4.60), we find the spectral density

$$\rho(p^2) = \theta(p^0)\theta(p^2)A(d_{\mathcal{U}_t})(p^2)^{d_{\mathcal{U}_t}-2}T^{\mu\nu\rho\sigma}(p) \tag{4.64}$$

In the case of symmetric and traceless unparticle operator $\mathcal{U}_\mu^\mu = 0$, $T^{\mu\nu\rho\sigma}(p)$ takes the form

$$T^{\mu\nu\rho\sigma}(p) = \frac{1}{2}\left(\Pi^{\mu\rho}(p)\Pi^{\nu\sigma}(p)+\Pi^{\mu\sigma}(p)\Pi^{\nu\rho}(p)-\frac{2}{3}\Pi^{\mu\nu}(p)\Pi^{\rho\sigma}(p)\right) \tag{4.65}$$

with $\Pi^{\mu\nu}(p) = -\eta^{\mu\nu}+\dfrac{p^\mu p^\nu}{p^2}$. Now, substituting the expression of $\rho(p^2)$ from eq(4.64) in the spectral representation (4.41) we find the propagator of tensor unparticles

$$\begin{aligned}\Delta_{\mathcal{U}_t}(p^2) &= \frac{1}{2\pi}\int dM^2\frac{\rho(M^2)}{p^2-M^2+i\epsilon}T^{\mu\nu\rho\sigma}(p)\\ &= \frac{1}{2\pi}\int dM^2\frac{A(d_{\mathcal{U}_t})(M^2)^{d_{\mathcal{U}_t}-2}}{p^2-M^2+i\epsilon}T^{\mu\nu\rho\sigma}(p)\\ &= \frac{A(d_{\mathcal{U}_t})}{2\sin(\pi d_{\mathcal{U}_t})}\frac{i}{(-p^2-i\epsilon)^{2-d_{\mathcal{U}_t}}}T^{\mu\nu\rho\sigma}(p)\end{aligned} \tag{4.66}$$

4.4 Deconstruction of unparticles

The unparticle fields as defined by Georgi are continuous distribution of massless states. Another representation of unparticles was introduced by Stephanov [32]. In his model, unparticle are represented as an infinite tower of massive particles with spacing Δ, which when it goes to zero $\Delta \to 0$, we recover the familiar unparticle representation.This deconstructed version of unparticles can be very useful in practical calculations (see for exemple ref [8]). Here we give a brief description of this model.

The correlation function of a scalar unparticle operator is

$$\int d^4xe^{ip\cdot x} \langle 0| TO(x)O^\dagger(0) |0\rangle = \int \frac{dM^2}{2\pi}\rho_O(M^2)\frac{i}{P^2 - M^2 + i} \tag{4.67}$$

scale invariance restrict the form of the spectral function ρ to be a power of M^2 as we have seen in previews sections

$$\rho_O(M^2) = A_{d_\mathcal{U}}(M^2)^{d_\mathcal{U}-2} \tag{4.68}$$

where $A_{d_\mathcal{U}}$ is the normalization constant defined in eq(2.11), and we also have

$$\rho_O(M^2) = 2\pi \sum_\lambda \delta(M^2 - M_\lambda^2) \langle 0| O(0) |\lambda\rangle \tag{4.69}$$

The sum is in fact an integral over the normalized states $|\lambda\rangle$. If we assume scale invariance to be broken in a controllable way, in place of a continuous spectrum of states $|\lambda\rangle$, we will have a discrete tower of states with the spacing between them controlled by the parameter Δ

$$M_n^2 = \Delta^2 n \tag{4.70}$$

If we introduce the matrix element

$$F_n^2 = |\langle 0| O(0) |\lambda_n\rangle|^2 \tag{4.71}$$

we can then write

$$\rho_O(M^2) = 2\pi \sum_n \delta(M^2 - M_n^2)F_n^2 \tag{4.72}$$

and the correlation function becomes

$$\int d^4xe^{ip\cdot x} \langle 0| TO(x)O^\dagger(0) |0\rangle = \sum_n \frac{iF_n^2}{p^2 - M_n^2 + i} \tag{4.73}$$

in the limit $\Delta \to 0$, the sum becomes an integral which must match the definition (4.67). So, we get

$$F_n^2 = \frac{A_{d_\mathcal{U}}}{2\pi}\Delta^2(M_n^2)^{d_\mathcal{U}-2} \tag{4.74}$$

4.5 Conformal symmetry breaking

Unparticles are conformal fields with a continuous mass distribution, but in reality we have not yet observed any fields with these characteristics. The absence of detection of scale invariant fields indicates that if unparticles exist in nature conformal symmetry must be broken at some energy scale. To implement conformal symmetry breaking (CSB) in the unparticle sector, the authors in [33] proposed the coupling of unparticle fields with Higgs boson as described by the following interaction term

$$\frac{1}{M^{d_{UV}-2}}|H|^2 O_{UV} \tag{4.75}$$

O_{UV} stands for the banks Zaks fields. At the energy scale Λ this interaction term flows to

$$C\frac{\Lambda^{d_{UV}-d_{IR}}}{M^{d_{UV}-2}}|H|^2 O_{IR} = |H|^2\mathcal{U} \tag{4.76}$$

where $\mathcal{U}$ is scalar unparticle field with scaling dimension $1 < d_{\mathcal{U}} = d_{IR} < 2$. The Higgs potential is as usual

$$V_0 = m^2|H|^2 + \lambda|H|^4 \tag{4.77}$$

Where m^2 is a term that can take either positive or negative values. When the Higgs field acquires VEV, via the normal process of electroweak symmetry breaking, it introduces a scale into the unparticle sector through the interaction term (4.75) which breaks the conformal invariance of the theory. In the language of renormalization group flows we say that the unparticle sector flows away from its conformal fixed point at an energy scale $\not{\Lambda}$. Below this scale the unparticle stuff behave like a traditional particle sector

$$\not{\Lambda}^{4-d_{\mathcal{U}}} = \left(\frac{\Lambda}{M}\right)^{d_{UV}-d_{\mathcal{U}}} M^{2-d_{\mathcal{U}}} v^2 \tag{4.78}$$

To keep the conformal window open, we must have $\Lambda > \not{\Lambda}$. This condition put a lower limit on the energy scale at which unparticle physics may be observed directly, which is consistent with experimental observations since no unparticle signals have yet been observed.

Since we de not know the details of CSB mechanism we can propose a simple model to incorporate scale invariance breaking into our theory. As is done in [5], we can adjust the spectral density (4.17) to be

$$\rho(p^2) = A_{d_{\mathcal{U}}}\theta(p^0)\theta(p^2 - m^2)(p^2 - m^2)^{d_{\mathcal{U}}-2} \tag{4.79}$$

where m^2 is the energy scale at which conformal invariance is broken. This term is equivalent to eliminating the mass modes of the unparticle spectrum inferior to m^2 which means that we introduced a mass gap into the theory. Another interesting proposal have been suggested by Delgado et al [34]. In their model the authors use a deconstructed version of unparticle fields (introduced in the previews section) to compute the VEV of the unparticle sector induced by its interaction with the Higgs field. In this case, there is an interplay between the unparticle sector and the Higgs sector in which the Higgs sector breaks conformal invariance of the unparticle sector and the unparticle sector, via the mixed term $vH\mathcal{U}$, shifts the VEV of the Higgs so the Higgs mass is no longer the familiar $\sqrt{2\lambda v^2}$ but it is defined as the pole of the spectral function constructed from mixed spectrum of the Higgs and unparticles.

The authors of [4] propose a slightly different interaction between the Higgs and unparticles. In this case, they imposed a parity symmetry on the unparticle fields to stabilize the vacuum so unparticles can be a viable candidate for dark matter. But the parity symmetry means that unparticle can only interact as pairs. So, the interaction term with the Higgs field is the following

$$L_{int} = \frac{c}{\Lambda^{2d_{\mathcal{U}}-2}} \mathcal{U}^2 \left(H^\dagger H\right) \tag{4.80}$$

4.6 Unparticle interactions

Unparticle interactions with SM particles are organized in the form of an effective interaction in accordance with the generic term

$$C \frac{\Lambda^{d_{BZ}-d_{SM}}}{M_{\mathcal{U}}^{d_{SM}+d_{BZ}-4}} O_{SM} \mathcal{U} \tag{4.81}$$

the term Λ serves as a cutuff energy scale to suppress interactions, so that the absence of detection of unparticles at low energy experiments, can be justified by the weakness of the coupling between the SM operators and unparticles. We can in principle construct different interaction terms in the form of (4.81) with scalar, vector, tensor and fermionic unparticles. Since in this chapter we only consider unparticle singlet under the SM gauge group the only symmetry that we must implement is Lorentz invariance. For scalar unparticles we have the following interactions [35, 36]

$$\begin{aligned}
&\lambda_0 \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \overline{f} f \mathcal{U} \\
&\lambda_0 \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \overline{f} i \gamma_5 f \mathcal{U} \\
&\lambda_0 \frac{1}{\Lambda^{d_{\mathcal{U}}}} \overline{f} \gamma_\mu \gamma_5 f \partial_\mu \mathcal{U} \\
&\frac{1}{\Lambda^{d_{\mathcal{U}}}} \left[\lambda_0 G_{\alpha\beta} G^{\alpha\beta} + \lambda_0^{'} G_{\alpha\beta} \tilde{G}^{\alpha\beta}\right] \mathcal{U}
\end{aligned} \tag{4.82}$$

$G^{\alpha\beta}$ is the gauge field strength tensor which can be gluons, photons, or weak gauge bosons and $\tilde{G}^{\alpha\beta}$ is given by

$$\tilde{G}^{\alpha\beta} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\mu\nu} \tag{4.83}$$

f is a standard model fermion which can be singlet or doublet. the term λ is a dimensionless coupling constant defined generically in eq(4.81). For tensor unparticles we have the following interactions

$$\begin{aligned}
&-\frac{1}{4} \lambda_2 \frac{1}{\Lambda^{d_{\mathcal{U}}}} \overline{f} i \left[\gamma_\mu \gamma_5 \overleftrightarrow{D}_\nu + \gamma_\nu \gamma_5 \overleftrightarrow{D}_\mu\right] f \mathcal{U}^{\mu\nu} \\
&\frac{1}{\Lambda^{d_{\mathcal{U}}}} \left[\lambda_2 G_{\mu\alpha} G^{\alpha}_{\nu} + \lambda_2^{'} G_{\mu\alpha} \tilde{G}^{\alpha}_{\nu}\right] \mathcal{U}^{\mu\nu}
\end{aligned} \tag{4.84}$$

where $D_\mu = \partial_\mu - ig \frac{\sigma^a}{2} W^a_\mu - ig' \frac{Y}{2} B_\mu$ is the covariant derivatives in the SM. For vector unparticles we have

$$\begin{aligned}
&\lambda_1 \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \overline{f} \gamma_\mu f \mathcal{U}^\mu \\
&\lambda_1 \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \overline{f} \gamma_\mu \gamma_5 f \mathcal{U}^\mu
\end{aligned} \tag{4.85}$$

fermionic unparticle interactions with SM particles are more stringent due to Lorentz invariance. We can only construct terms of the form

$$\overline{f} \gamma_\mu \mathcal{U}_f A^\mu \tag{4.86}$$

here A^μ is a standard model gauge boson.

4.7 The AdS/CFT correspondence

Since the paper by Maldacena [37] that established a correspondence between a $5d$ gravity in the anti de sitter space (AdS) and a $4d$ CFT theory on the boundary, a lot of effort have been devoted to holographic models in many area of theoretical physics [38]. The advantage of this approach is that problems which are very difficult typically in CFT are easier to solve on the gravity side and vise versa. The unparticle model, which is a CFT theory is no exception. The use of the correspondence to better understand unparticle physics is still a work in progress. Authors like Cacciapaglia et al have found a way to use this duality to deal with the limitation of the effective lagrangian description of unparticles. For example in the paper [5] it was shown that cross section for two particle scattering mediated by charged scalar unparticle takes negative value for scale dimension $d > 2$, but in [39] the case for $d > 2$ was treated successfully using the correspondence. However, despite the advantages of the AdS/CFT correspondence, there are considerable challenges, particularly that not all CFT have AdS duals, and in case they do, finding them is not a trivial task. As Georgy himself said, "while Ads based models can provide very useful guidance and examples, their ability to describe realistic unparticle physics scenarios is limited" [40]

Chapter 5

Gauged unparticles

The nontrivial form of the unparticle propagator, with non integer scaling dimension d, leads to nonlocal action for unparticles which make a gauge invariant formalism harder to construct. In this case, the minimal coupling prescription used in the SM is no longer valid. Fortunately, a similar problem has been solved by Terning [41] et al and Holdom [42] in the context of non local chiral quark model. Based on that work the first gauged unparticle model was constructed by Cacciapaglia et al [5]. Later works in this subject, based on different approches, subsequently followed [43, 44, 9]. In this chapter we follow the work of [5].

The presence of unparticle that carries SM quantum numbers can modify drastically the low energy phenomenology, including electroweak precision observables, which are in excellent agreement with the SM predictions. For this reason conformal symmetry must be broken, so that unparticles fields don't appear at low energies in contradiction with experiments. One way to implement conformal symmetry breaking is to modify the unparticle propagator with an IR cut-off m, which parametrize our ignorance about the details of conformal symmetry breaking. In this case the spectral representation of scalar unparticles is

$$\begin{aligned}\Delta_s(p,m) &= \int d^4x e^{ip\cdot x} \langle 0| T\mathcal{U}_s(x)\mathcal{U}_s^\dagger(0) |0\rangle \\ &= \frac{A_{d_{\mathcal{U}_s}}}{2\pi}\int_{m^2}^{\infty} dM^2 \left(M^2-m^2\right)^{d_{\mathcal{U}_s}-2} \frac{i}{p^2-M^2+i\epsilon}\end{aligned} \tag{5.1}$$

performing the integration in (5.1) we find

$$\Delta_s(p,m) = \frac{A_{d_{\mathcal{U}_s}}}{2\sin(\pi d_{\mathcal{U}_s})}\frac{i}{\Sigma_s(p^2,m^2)} \tag{5.2}$$

where $\Sigma_s(p^2, m^2) = (m_s^2 - p^2)^{2-d_{\mathcal{U}_s}}$. In the limit $d_{\mathcal{U}_s} \to 1$ this propagator reduces to a propagator for free massive scalar field. The effective action corresponding to the propagator (5.2) is the following

$$\begin{aligned} S &= \int \frac{d^4p}{(2\pi)^4} \mathcal{U}_s^\dagger(p) \tilde{\Delta}_s^{-1}(p,m) \mathcal{U}_s(p) \\ &= \frac{2\sin(\pi d_{\mathcal{U}_s})}{A_{d_{\mathcal{U}_s}}} \int \frac{d^4p}{(2\pi)^4} \mathcal{U}_s^\dagger(p) \left(m^2 - p^2\right)^{2-d_{\mathcal{U}_s}} \mathcal{U}_s(p) \end{aligned} \tag{5.3}$$

If we Fourier transform the action (5.3) we arrive at a non local action in the configuration space

$$S = \int d^4x d^4y \mathcal{U}_s^\dagger(x) \Delta_s^{-1}(x-y) \mathcal{U}_s(y) \tag{5.4}$$

where $\Delta_s^{-1}(z)$ is the Fourier transform of $\tilde{\Delta}_s^{-1}(p,m)$

$$\Delta_s^{-1}(z) = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot z} \tilde{\Delta}_s^{-1}(p,m) \tag{5.5}$$

To insure gauge invariance we introduce a Wilson line W [45] between the two unparticle fields

$$S = \int d^4x d^4y \mathcal{U}_s^\dagger(x) \Delta_s^{-1}(x-y) W(x,y) \mathcal{U}_s(y) \tag{5.6}$$

where

$$W(x,y) = P \exp\left(-ig_s T^a \int_x^y A_\mu^a(u) du^\mu\right) \tag{5.7}$$

where T^a are the generators of the SM groups $SU(3)_C$, $SU(2)_L$ or $U(1)_Y$. The vectors fields A_μ^a are the corresponding gauge bosons. P is the path ordering operators for the fields A_μ^a between the points x and y. The Wilson line (5.7) obeys the Mandelstam condition, applied by that author to quantum electrodynamics, as follows [46]

$$\frac{\partial}{\partial x^\mu} W(x,y) = igT^a A_\mu^a W(x,y) \tag{5.8}$$

a similar condition holds for y. The action (5.6) is invariant under the following gauge transformations

$$\mathcal{U}_s \to U^{-1}\mathcal{U}_s,\ \mathcal{U}_s^\dagger \to \mathcal{U}_s^\dagger U,\ A_\mu = T^a A_\mu^a \to U^{-1} A_\mu U + \frac{1}{ig} U^{-1} \partial_\mu U \tag{5.9}$$

as the Wilson line transforms as

$$W(x,y) \to U^{-1}(x)W(x,y)U(y) \tag{5.10}$$

Now, using the same techniques developed by Terning et al, we can derive the vertex functions describing the interactions between unparticle fields and any number of gauge bosons A_μ. For the propose of our work, we only focus on the coupling between unparticles and one ($A^a_\mu \mathcal{U}_s \mathcal{U}^\dagger_s$)or two gauge bosons ($\mathrm{A}^a_\mu A^b_\nu \mathcal{U}_s \mathcal{U}^\dagger_s$) as follows

$$\begin{aligned} ig_s\Gamma^{a\mu} &= \frac{i\delta^3 S}{\delta A^{a\mu}(q)\delta\mathcal{U}^\dagger_s(p+q)\mathcal{U}_s(p)} \\ &= ig_s T^a(2p^\mu + q^\mu)\Sigma^s_1(p,q) \\ &= ig_s T^a(2p^\mu + q^\mu)\frac{2\sin(\pi d_{\mathcal{U}_s})}{A_{d_{\mathcal{U}_s}}}\frac{\left[(m^2-(p+q)^2)^{2-d_{\mathcal{U}_s}} - (m^2-p^2)^{2-d_{\mathcal{U}_s}}\right]}{2p\cdot q + q^2} \end{aligned} \tag{5.11}$$

we can check that this vertex satisfies the Ward-Takahashi identity [47, 48]

$$iq_\mu\Gamma^{a\mu} = \Delta^{-1}_s(p+q,m)T^a - T^a\Delta^{-1}_s(p,m) \tag{5.12}$$

For the coupling between unparticles and two gauge bosons we have

$$\begin{aligned} ig^2_s\Gamma^{ab\mu\nu} &= \frac{i\delta^3 S}{\delta A^{a\mu}(q_1)\delta A^{a\nu}(q_2)\delta\mathcal{U}^\dagger_s(p+q_1+q_2)\mathcal{U}_s(p)} \\ &= ig^2_s\Big[\{T^a,T^b\}\eta^{\mu\nu}\Sigma^s_1(p,q_1+q_2) + T^aT^b(2p+q_2)^\nu(2p+2q_2+q_1)^\mu \\ &\times \Sigma^s_2(p,q_2,q_1) + T^bT^a(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma^s_2(p,q_1,q_2)\Big] \end{aligned} \tag{5.13}$$

where the form factors Σ^s_1 and Σ^s_2 are given by

$$\begin{aligned} \Sigma^s_1(p,q) &= \frac{2\sin(\pi d_{\mathcal{U}_s})}{A_{d_{\mathcal{U}_s}}}\frac{\left[(m^2-(p+q)^2)^{2-d_{\mathcal{U}_s}} - (m^2-p^2)^{2-d_{\mathcal{U}_s}}\right]}{2p\cdot q + q^2} \\ &= \frac{2\sin(\pi d_{\mathcal{U}_s})}{A_{d_{\mathcal{U}_s}}}\frac{\Sigma^s_0(p+q) - \Sigma^s_0(p)}{(p+q)^2 - p^2} \end{aligned} \tag{5.14}$$

$$\Sigma^s_2(p,q_1,q_2) = \frac{\Sigma^s_1(p,q_1+q_2) - \Sigma^s_1(p,q_1)}{(p+q_1+q_2)^2 - (p+q_1)^2} \tag{5.15}$$

In the canonical limit, $d_{\mathcal{U}_s} \to 1$, eq(5.11) and (5.13) reduce to the usual coupling between standard scalar fields and gauge bosons

$$ig_s\Gamma^{a\mu} = ig_s T^a(2p+q)^\mu \tag{5.16}$$

$$ig_s^2\Gamma^{ab\mu\nu} = ig_s^2\left(T^aT^b + T^bT^a\right) \tag{5.17}$$

Now, we repeat the same process for fermionic unparticles. The propagator of unfermion is

$$\begin{aligned}\Delta_f(p,m) &= \frac{A_{d_{\mathcal{U}_f}}}{2\pi}\int_{m^2}^{\infty} dM^2(M^2-m^2)^{5/2-d_{\mathcal{U}_f}}\frac{i\not{p}}{p^2-M^2+i\epsilon}\\ &= \frac{A_{d_{\mathcal{U}_f}}}{2\cos\left(d_{\mathcal{U}_f}\pi\right)}\frac{i}{(\not{p}-m)\Sigma_0^f(p)}\end{aligned} \tag{5.18}$$

where $\Sigma_0^f(p) = (m^2-p^2)^{3/2-d_{\mathcal{U}_f}}$. The corresponding effective action is

$$\begin{aligned}S &= \int\frac{d^4p}{(2\pi)^4}\mathcal{U}_f^\dagger(p)\tilde{\Delta}_f^{-1}(p,m)\mathcal{U}_f(p)\\ &= -\frac{2\cos\left(d_{\mathcal{U}_f}\pi\right)}{A_{d_{\mathcal{U}_f}}}\int\frac{d^4p}{(2\pi)^4}\overline{\mathcal{U}}_f(p)\frac{(m^2-p^2)^{5/2-d_{\mathcal{U}_f}}}{\not{p}+m}\mathcal{U}_f(p)\\ &= \frac{2\cos\left(d_{\mathcal{U}_f}\pi\right)}{A_{d_{\mathcal{U}_f}}}\int\frac{d^4p}{(2\pi)^4}\overline{\mathcal{U}}_f(p)(m^2-p^2)^{3/2-d_{\mathcal{U}_f}}(\not{p}-m)\mathcal{U}_f(p)\\ &= \frac{2\cos\left(d_{\mathcal{U}_f}\pi\right)}{A_{d_{\mathcal{U}_f}}}\int\frac{d^4p}{(2\pi)^4}\overline{\mathcal{U}}_f(p)\Sigma_0^f(p)(\not{p}-m)\mathcal{U}_f(p)\end{aligned} \tag{5.19}$$

To implement gauge symmetry we Fourier transform the action (5.19) into a no local action in configuration space and introduce a Wilson line as follows

$$S = \frac{2\cos\left(d_{\mathcal{U}_f}\pi\right)}{A_{d_{\mathcal{U}_f}}}\int d^4xd^4y\overline{\mathcal{U}}_f(x)\left(-i\not{\partial}_y\Sigma_0^f(x-y)\right)W(x,y)\mathcal{U}_f(y) \tag{5.20}$$

where $W(x,y)$ is the same as (5.7).

Now, we derive the vertex functions for the coupling of unfermion with one and two gauge bosons by taking functional derivative of the appropriate fields. The coupling between two unfermion and one gauge boson is given by

$$\begin{aligned}ig_f\Gamma^{a\mu} &= \frac{i\delta^3S}{\delta A^{a\mu}(q)\delta\overline{\mathcal{U}}_f(p+q)\mathcal{U}_f(p)}\\ &= i\frac{g_f}{2}\left\{\gamma^\mu T^a\left[\Sigma_0^f(p+q)+\Sigma_0^f(p)\right] + (2\not{p}+\not{q}-2m)\,T^a(2p+q)^\mu\Sigma_1^f(p,q)\right\}\end{aligned} \tag{5.21}$$

where the form factor Σ_1^f is the following

$$\Sigma_1^f(p,q) = \frac{\left[(m^2-(p+q)^2)^{3/2-d_{\mathcal{U}_f}} - (m^2-p^2)^{3/2-d_{\mathcal{U}_f}}\right]}{2p\cdot q+q^2} \tag{5.22}$$

One can check that the vertex (5.21) satisfies the Ward-Takahachi Identity

$$iq_\mu \Gamma^{a\mu} = \left[\Delta_f^{-1}(p+q,m) - \Delta_f^{-1}(p,m)\right] T^a \tag{5.23}$$

The coupling between two unfermions and two gauge bosons is given by

$$\begin{aligned}
ig_f^2 \Gamma^{ab\mu\nu} &= \frac{i\delta^3 S}{\delta A^{a\mu}(q_1)\delta A^{b\nu}(q_2)\delta\overline{\mathcal{U}}_f(p+q_1+q_2)\mathcal{U}_f(p)} \\
&= i\frac{g_f^2}{2}\Big\{ (2\not{p}+\not{q_1}+\not{q_2}-2m)\left[\{T^a,T^b\}\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2) + T^aT^b(2p+q_2)^\nu\right. \\
&\times (2p+2q_2+q_1)^\mu \Sigma_2^f(p,q_2,q_1) + T^bT^a(2p+q_1)^\mu(2p+2q_1+q_2)^\mu \\
&\left.\times\, \Sigma_2^f(p,q_1,q_2)\right] + \gamma^\mu \Gamma_f^{ab\nu}(p,q_2,q_1) + \gamma^\nu \Gamma_f^{ab\mu}(p,q_1,q_2)\Big\}
\end{aligned} \tag{5.24}$$

where the form factor Σ_2^f is given by

$$\Sigma_2^f(p,q_1,q_2) = \frac{\Sigma_1^f(p,q_1+q_2) - \Sigma_1^f(p,q_1)}{(p+q_1+q_2)^2-(p+q_1)^2} \tag{5.25}$$

and $\Gamma_f^{ab\mu}$ is the following

$$\Gamma_f^{ab\mu}(p,q_1,q_2) = T^aT^b(2p+q_1)^\mu\Sigma_1^f(p,q_2) + T^bT^a(2p+2q_2+q_1)^\mu\Sigma_1^f(p+q_2,q_1) \tag{5.26}$$

In the canonical limit, $d_{\mathcal{U}_f} \to 3/2$, we recover the usual vertex of fermion-gauge bosons couplings

$$ig_f\Gamma_f^{a\mu} \to ig_f T^a\gamma^\mu \tag{5.27}$$

$$ig_f^2\Gamma_f^{ab\mu\nu} \to 0 \tag{5.28}$$

The second vertex vanishes in ($d_{\mathcal{U}_f} \to 3/2$) which means that the coupling between two unfermion and two gauge bosons is a novel interaction for unparticle physics(there is no equivalent interaction in the SM).

5.1 Unparticle coupling with SM electroweak gauge bosons

In our work, we will need to derive expressions for unparticle coupling with SM electroweak gauge bosons W, Z and γ. For this purpose, we derive in this section the vertices that enter in the computation of polarization functions $\Pi^{\gamma\gamma}$, Π^{ZZ}, $\Pi^{\gamma Z}$, and Π^{WW}. We will use the results to calculate the oblique parameters and the evolution of the running coupling induced by virtual scalar unparticles that carries both the color charge and electroweak quantum numbers.

We start from the action

$$S = \int d^4x d^4y \Big(\mathcal{U}_L^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) \mathcal{W}_L(x,y) \mathcal{U}_L(y) + \mathcal{U}_R^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) \mathcal{W}_R(x,y) \mathcal{U}_R(y) \Big) \tag{5.29}$$

here we suppose that unparticles carries weak quantum numbers. $\mathcal{U}_L$ is a left handed unfermion doublet that transform under the group $SU(2)_L$ and $\mathcal{U}_R$ is a singlet under the group $SU(2)$, but transform according to the hypercharge group $U(1)_Y$.

To insure gauge invariance we have introduced the Wilson lines $\mathcal{W}_{L,R}$ defined as

$$\mathcal{W}_L(x,y) = P \exp\Big(\int_x^y (-igT^a W_\mu^a(u) - ig^{'} Y B_\mu(u)) du^\mu \Big) \tag{5.30}$$

$$\mathcal{W}_R(x,y) = \exp\Big(\int_x^y -ig^{'} Q B_\mu(u) du^\mu \Big) \tag{5.31}$$

where as usual P denotes path ordering that effects on the generators T^a in the unparticle representation. Q is the charge operator defined in the same representation. g, g' are the gauge coupling of the SM groups $SU(2)_L$ and $U(1)_Y$ respectively. W_μ^a are the weak gauge bosons associated with the three generators of the isospin group $SU(2)_L$ and B_μ is the gauge boson associated with the group U_Y. To find the interaction vertices of unparticles with the physical gauge bosons Z, W and γ we replace W_μ^a, B_μ in eq(5.30) and eq(5.31) according to the relations

$$W_3^\mu = \cos(\theta_W) Z^\mu + \sin(\theta_W) A^\mu \tag{5.32}$$

$$B^\mu = -\sin(\theta_W) Z^\mu + \sin(\theta_W) A^\mu \tag{5.33}$$

$$W^\mu = (W_1 + iW_2)/\sqrt{2}, W^{\mu\dagger} = (W_1 - iW_2)/\sqrt{2} \tag{5.34}$$

where θ_W is the Weinberg mixing angle. Substituting in the action (5.29) we get

$$S = \int d^4x d^4y \Big\{ \mathcal{U}_L^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) P \exp\Big(-ig \int_x^y \frac{1}{2}(\sigma^- W_\mu^-(u) + \sigma^+ W_\mu^+(u)) du^\mu - \int_x^y i(g\frac{\sigma_3}{2}\cos(\theta_W) + g' Y_1 \sin(\theta_W)) Z_\mu(u) du^\mu - e\int_x^y \Big(\frac{\sigma_3}{2} + Y_1\Big) A_\mu(u) du^\mu \Big) \mathcal{U}_L + \mathcal{U}_R^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) \exp\Big(+ig' \int_x^y \sin(\theta_W) Q Z_\mu(u) du^\mu + ie \int_x^y Q A_\mu(u) du^\mu \Big) \mathcal{U}_R \Big\} \tag{5.35}$$

where $\mathcal{U}_L, \mathcal{U}_R$ are defined as

$$\begin{aligned}\mathcal{U}_L &= \frac{1-\gamma_5}{2}\mathcal{U} \\ \mathcal{U}_R &= \frac{1+\gamma_5}{2}\mathcal{U}\end{aligned} \tag{5.36}$$

Substituting the last two expressions in eq(5.35) we find

$$\begin{aligned}S = \int d^4x d^4y \Big\{ &\mathcal{U}^\dagger(x)\tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) P \exp\Big(-ig\int_x^y \frac{1}{2}(\sigma^- W_\mu^-(u) + \sigma^+ W_\mu^+(u)) du^\mu \\ &- \int_x^y i(g\frac{\sigma_3}{2}\cos(\theta_W) + g' Y_1 \sin(\theta_W)) Z_\mu(u) du^\mu - e\int_x^y \left(\frac{\sigma_3}{2} + Y_1\right) A_\mu(u) du^\mu \Big) \frac{1-\gamma_5}{2}\mathcal{U}(y) \\ &+ \mathcal{U}^\dagger(x)\tilde{\Delta}_{\mathcal{U}}^{-1}(x-y)\exp\Big(+ig'\int_x^y \sin(\theta_W) Q Z_\mu(u) du^\mu + ie\int_x^y Q A_\mu(u) du^\mu \Big) \frac{1+\gamma_5}{2}\mathcal{U}(y) \Big\}\end{aligned} \tag{5.37}$$

where $\mathcal{U} = \mathcal{U}_L + \mathcal{U}_R$ is the parity conserving fermionic unparticle. Now, using functional derivatives with respect to the appropriate fields we find the coupling of unparticles to one photon

$$\begin{aligned} ig_f\Gamma^{a\mu} &= \frac{i\delta^3 S}{\delta A^{a\mu}(q)\delta\mathcal{U}_f^\dagger(p+q)\mathcal{U}_f(p)} \\ &= i\frac{eQ}{2}\left\{ \gamma^\mu \left[\Sigma_0^f(p+q) + \Sigma_0^f(p)\right] \frac{1-\gamma_5}{2} + (2\not{p} + \not{q} - 2m)(2p+q)^\mu \Sigma_1^f(p,q)\frac{1-\gamma_5}{2} \right\} \\ &+ i\frac{eQ}{2}\left\{ \gamma^\mu \left[\Sigma_0^f(p+q) + \Sigma_0^f(p)\right] \frac{1+\gamma_5}{2} + (2\not{p} + \not{q} - 2m)(2p+q)^\mu \Sigma_1^f(p,q)\frac{1+\gamma_5}{2} \right\} \\ &= i\frac{eQ}{2}\left\{ \gamma^\mu \left[\Sigma_0^f(p+q) + \Sigma_0^f(p)\right] + (2\not{p} + \not{q} - 2m)(2p+q)^\mu \Sigma_1^f(p,q) \right\} \end{aligned} \tag{5.38}$$

this vertex is equal to the one described in eq(5.21) if we make the following substitutions

$$g_f = e,\ T^a = Q \tag{5.39}$$

If we take the canonical limit $d_{\mathcal{U}_f} \to \frac{3}{2}$, $ig_f\Gamma^{a\mu}$ becomes

$$ig_f\Gamma^{a\mu} = ieQ\gamma^\mu \tag{5.40}$$

which is exactly the vertex describing the interaction between an ordinary fermion and a photon.

The coupling between two unfermion and two photon is found to be

$$
\begin{aligned}
ig_f^2\Gamma^{ab\mu\nu} \quad &= \frac{i\delta^3 S}{\delta A^{a\mu}(q_1)\delta A^{b\nu}(q_2)\delta\overline{\mathcal{U}}_f(p+q_1+q_2)\mathcal{U}_f(p)} \\
&= i\frac{e^2}{2}\Big\{ (2\not{p}+\not{q}_1+\not{q}_2-2m)\,\frac{1-\gamma_5}{2}\big[2Q^2\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+Q^2(2p+q_2)^\nu \\
&\times 2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1)+Q^2(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\big] \\
&+\gamma^\mu\frac{1-\gamma_5}{2}\Gamma_f^{ab\nu}(p,q_2,q_1)+\gamma^\nu\frac{1-\gamma_5}{2}\Gamma_f^{ab\mu}(p,q_1,q_2)\Big\} \\
&+i\frac{e^2}{2}\Big\{ (2\not{p}+\not{q}_1+\not{q}_2-2m)\,\frac{1+\gamma_5}{2}\big[2Q^2\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+Q^2(2p+q_2)^\nu \\
&2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1)+Q^2(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\big] \\
&+\gamma^\mu\frac{1+\gamma_5}{2}\Gamma_f^{ab\nu}(p,q_2,q_1)+\gamma^\nu\frac{1+\gamma_5}{2}\Gamma_f^{ab\mu}(p,q_1,q_2)\Big\}
\end{aligned}
\tag{5.41}
$$

after simplification we find

$$
\begin{aligned}
ig_f^2\Gamma^{ab\mu\nu} = i\frac{e^2}{2}\Big\{ &(2\not{p}+\not{q}_1+\not{q}_2-2m)\,\big[2Q^2\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+Q^2(2p+q_2)^\nu \\
&\times 2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1)+Q^2(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\big] \\
&+\gamma^\mu\Gamma_f^{ab\nu}(p,q_2,q_1)+\gamma^\nu\Gamma_f^{ab\mu}(p,q_1,q_2)\Big\}
\end{aligned}
\tag{5.42}
$$

where the form factors Σ_1^f, Σ_2^f are the same functions defined in the previews section eqs(5.22,5.25). $\Gamma_f^{ab\mu}$ is given by

$$
\Gamma_f^{ab\mu} = Q^2\Big((2p+q_2)^\mu\Sigma_1^f(p,q_1)+(2p+2q_1+q_2)^\mu\Sigma_1^f(p+q_1,q_2)\Big) \tag{5.43}
$$

Repeating the same procedure we find the coupling between unfermion and Z bosons. The $\mathcal{U}^\dagger\mathcal{U}Z^\mu$ vertex is

$$
\begin{aligned}
ig_f\Gamma^{a\mu} &= \frac{i\delta^3 S}{\delta Z^{a\mu}(q)\delta\mathcal{U}_f^\dagger(p+q)\mathcal{U}_f(p)} \\
&= i\frac{g\frac{\sigma_3}{2}-g'Y\sin(\theta_W)}{2}\Big\{\gamma^\mu\Big[\Sigma_0^f(p+q)+\Sigma_0^f(p)\Big]\,\frac{1-\gamma_5}{2}+(2\not{p}+\not{q}-2m)\,(2p+q)^\mu \\
&\times\Sigma_1^f(p,q)\frac{1-\gamma_5}{2}\Big\}-i\frac{g'\sin(\theta_W)Q}{2}\Big\{\gamma^\mu\Big[\Sigma_0^f(p+q)+\Sigma_0^f(p)\Big]\,\frac{1+\gamma_5}{2} \\
&+(2\not{p}+\not{q}-2m)\,(2p+q)^\mu\Sigma_1^f(p,q)\frac{1+\gamma_5}{2}\Big\}
\end{aligned}
\tag{5.44}
$$

using the relation $e = g\sin(\theta_W) = g'\cos(\theta_W)$ and the Gell-Mann–Nishijima formula $Y = Q - \frac{\sigma_3}{2}$ we find

$$ig_f\Gamma^{a\mu} = i\frac{e}{4\sin(\theta_W)\cos(\theta_W)}\Big\{\gamma^\mu\left[\Sigma_0^f(p+q)+\Sigma_0^f(p)\right]\left(\frac{\sigma_3}{2}-2\sin(\theta_W)^2Q-\frac{\sigma_3}{2}\gamma_5\right) + (2\not{p}+\not{q}-2m)\left(\frac{\sigma_3}{2}-2\sin(\theta_W)^2Q-\frac{\sigma_3}{2}\gamma_5\right)(2p+q)^\mu\Sigma_1^f(p,q)\Big\} \tag{5.45}$$

If we take the limit $d_{\mathcal{U}_f}\to\frac{3}{2}$, this vertex reduces to

$$ig_f\Gamma^{a\mu} = i\frac{e}{2\sin(\theta_W)\cos(\theta_W)}\gamma^\mu\left(\frac{\sigma_3}{2}-2\sin(\theta_W)^2Q-\frac{\sigma_3}{2}\gamma_5\right) \tag{5.46}$$

this expression is the same vertex describing the the interaction between an ordinary fermion and a Z boson in the SM.
The vertex describing the interaction between two unfermions and two Z bosons is the following

$$\begin{aligned} ig_f^2\Gamma^{ab\mu\nu} &= \frac{i\delta^3 S}{\delta Z^{a\mu}(q_1)\delta Z^{b\nu}(q_2)\delta\overline{\mathcal{U}}_f(p+q_1+q_2)\mathcal{U}_f(p)} \\ &= i\frac{e^2}{4\sin(\theta_W)^2\cos(\theta_W)^2} \\ &\times\Big\{(2\not{p}+\not{q_1}+\not{q_2}-2m)\left(\frac{\sigma_3^2}{4}+2\sin(\theta_W)^4Q^2-\sin(\theta_W)^2\sigma_3Q-\gamma_5\left(\sigma_3^24-\sin(\theta_W)^2\sigma_3Q\right)\right) \\ &\times\Big[\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+(2p+q_2)^\nu(2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1)+(2p+q_1)^\mu(2p+2q_1+q_2)^\mu \\ &\times\Sigma_2^f(p,q_1,q_2)\Big]+\gamma^\mu\Gamma_f^{ZZ\nu}(p,q_2,q_1)+\Gamma_f^{ZZ\mu}(p,q_1,q_2)\Big\} \end{aligned} \tag{5.47}$$

where the form factor $\Gamma_f^{ZZ\mu}$ is defined as

$$\begin{aligned}\Gamma_f^{ZZ\mu}(p,q_2,q_1) &= \left(\frac{\sigma_3^2}{4}+2\sin(\theta_W)^4Q^2-\sin(\theta_W)^2\sigma_3Q-\gamma_5\left(\frac{\sigma_3^2}{4}-\sin(\theta_W)^2\sigma_3Q\right)\right) \\ &\times\left((2p+q_2)^\mu\Sigma_1^f(p,q_1)+(2p+2q_1+q_2)^\mu\Sigma_1^f(p+q_1,q_2)\right)\end{aligned} \tag{5.48}$$

in the same manner we derive the vertex functions for weak gauge bosons

W, $W^\dagger$. For the vertex $\mathcal{U}^\dagger\mathcal{U}W^\mu$ we have

$$\begin{aligned} ig_f\Gamma^{a\mu} &= \frac{i\delta^3 S}{\delta W^{a\mu}(q)\delta\mathcal{U}_f^\dagger(p+q)\mathcal{U}_f(p)} \\ &= i\frac{e}{4\sin(\theta_W)}\sigma^-\left\{\gamma^\mu\left[\Sigma_0^f(p+q)+\Sigma_0^f(p)\right]+(2\not{p}+\not{q}-2m)\,(2p+q)^\mu\Sigma_1^f(p,q)\right\}\frac{1-\gamma_5}{2} \end{aligned} \quad (5.49)$$

for the vertex $\mathcal{U}^\dagger\mathcal{U}W^\mu W^\nu$ we have the following result

$$\begin{aligned} ig_f^2\Gamma^{ab\mu\nu} &= \frac{i\delta^3 S}{\delta W^{a\mu}(q_1)\delta W^{b\nu}(q_2)\delta\overline{\mathcal{U}}_f(p+q_1+q_2)\mathcal{U}_f(p)} \\ &= i\frac{e^2}{8\sin(\theta_W)^2}\Big\{\,(2\not{p}+\not{q}_1+\not{q}_2-2m)\,\big[\{\sigma^-,\sigma^+\}\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+\sigma^-\sigma^+(2p+q_2)^\nu \\ &\times 2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1)+\sigma^+\sigma^-(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\big] \\ &+\gamma^\mu\Gamma_f^{WW\nu}(p,q_2,q_1)+\gamma^\nu\Gamma_f^{WW\mu}(p,q_1,q_2)\Big\}\frac{1-\gamma_5}{2} \end{aligned} \quad (5.50)$$

where the form factor $\Gamma_f^{WW\mu}$ is given by

$$\Gamma_f^{WW\mu}=\sigma^-\sigma^+(2p+q_1)^\mu\Sigma_1^f(p,q_2)+\sigma^+\sigma^-(2p+2q_2+q_1)^\mu\Sigma_1^f(p+q_2,q_1) \quad (5.51)$$

here σ^-, σ^+ are the lowering and raising operators, receptively, of the the $SU(2)$ group defined by

$$\sigma^-=\frac{\sigma_1-i\sigma_2}{\sqrt{2}},\ \sigma^+=\frac{\sigma_1+i\sigma_2}{\sqrt{2}} \quad (5.52)$$

where σ_1, σ_2 are Pauli matrices. Finally, we derive the vertex function describing the coupling between two unfermion, one photon and one Z boson $\mathcal{U}^\dagger\mathcal{U}Z^\mu A^\nu$ as follows

$$\begin{aligned} ig_f^2\Gamma^{ab\mu\nu} &= \frac{i\delta^3 S}{\delta Z^{a\mu}(q_1)\delta A^{b\nu}(q_2)\delta\overline{\mathcal{U}}_f(p+q_1+q_2)\mathcal{U}_f(p)} \\ &= i\frac{e^2}{4\sin(\theta_W)\cos(\theta_W)} \\ &\times\Big\{\,(2\not{p}+\not{q}_1+\not{q}_2-2m)\,\Big[\eta^{\mu\nu}\Sigma_1^f(p,q_1+q_2)+(2p+q_2)^\nu(2p+2q_2+q_1)^\mu\Sigma_2^f(p,q_2,q_1) \\ &+(2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\Big]+\gamma^\mu\Gamma_f^{ZZ\nu}(p,q_2,q_1)+\gamma^\nu\Gamma_f^{ZZ\mu}(p,q_1,q_2)\Big\} \\ &\times\left(\frac{\sigma_3}{2}-\gamma_5\left(\sigma_3 2-2\sin^2(\theta_W)Q\right)\right) \end{aligned} \quad (5.53)$$

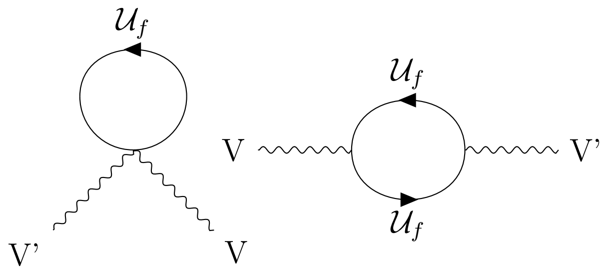

Figure 5.1: The one loop contribution to polarization functions from charged fermionic unparticle fields, V and V' stand for γ, Z or W.

Taking a closer look at the structure of the previews vertices, we can give a general form for the interactions between one gauge boson and two unfermion as follows,

$$\begin{aligned} ig_a\Gamma^{a\mu} = ig_a\Big\{&\gamma^\mu(T_a + T_b\gamma_5)\left[\Sigma_0^f(p+q) + \Sigma_0^f(p)\right] + (2\not p + \not q - 2m)\,(T_a + T_b\gamma_5) \\ &\times (2p+q)^\mu \Sigma_1^f(p,q)\Big\} \end{aligned} \tag{5.54}$$

in the same manner we can generalize the expression for the coupling between two gauge bosons and two unfermion with the general expression

$$\begin{aligned} ig_a g_b\Gamma^{ab\mu\nu} = ig_a g_b\Big\{&\,(2\not p + \not q_1 + \not q_2 - 2m)\,(T_1 + T_2\gamma_5)\big[\eta^{\mu\nu}\Sigma_1^f(p, q_1+q_2) + (2p+q_2)^\nu \\ &\times 2p + 2q_2 + q_1)^\mu \Sigma_2^f(p,q_2,q_1) + (2p+q_1)^\mu(2p+2q_1+q_2)^\mu\Sigma_2^f(p,q_1,q_2)\big] \\ &+ \gamma^\mu(T_1+T_2\gamma_5)\Gamma_f^{ab\nu}(p,q_2,q_1) + \gamma^\nu(T_1+T_2\gamma_5)\Gamma_f^{ab\mu}(p,q_1,q_2)\Big\} \end{aligned} \tag{5.55}$$

5.2 Polarization functions for unfermion

Now that we have derived Feynman vertices for the interaction between unparticles and electroweak gauge bosons, we can calculate the unparticle contribution to the polarization functions $\Pi^{ab\mu\nu}$. Later we will use these functions to compute the fermionic unparticle contribution to the oblique parameters. In Fig (5.1), we show a typical diagram of the fermionic unparticles loops contributions to the self-energy functions $\Pi^{ab\mu\nu}$ at the one loop level, where V and V' stand for γ, Z or W.

The expression for the diagram in the right hand side (a) is given by

$$\Pi_{(a)}^{ab\mu\nu} = -\mu^{4-D}\int \frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left(\Gamma^{a\mu}(p,q,p+q)S(p)\Gamma^{b\nu}(p+q,-q,-p)S(p+q)\right) \tag{5.56}$$

For the diagram on the left hand side (b) we have

$$\Pi^{ab\mu\nu}_{(b)} = -\mu^{4-D} \int \frac{d^D p}{(2\pi)^D} \operatorname{Tr}\left(\Gamma^{ab\mu\nu}(p, q, p+q) S(p)\right) \tag{5.57}$$

where $\Gamma^{a\mu}(p, q, p+q)$ is the three point vertex function (5.54), $\Gamma^{ab\mu\nu}(p, q, p+q)$ is the four point vertex function (5.55), $S(p)$ is the unfermion propagator in momentum space and μ is the renormalization constant. The complicated Feynman vertices and the non standard form of the propagator (5.18) do not allow the application of the usual tensor reduction recipe to compute the integrals (5.56) and (5.57). However, If we look at the large p region of the loop integrals, we can affect a Taylor expansion of the functions $\Sigma^f_0(p+q)$ for small q, this is allowed because the beta function of the theory is sensible only to the UV regime [49]. We begin by writing $\Sigma^f_0(p)$ as follows

$$\Sigma^f_0(p) = \left(m^2 - (p)^2\right)^{3/2-d_{\mathcal{U}_f}} = (-1)^{3/2-d_{\mathcal{U}_f}} \left((p)^2 - m^2\right)^{3/2-d_{\mathcal{U}_f}} \tag{5.58}$$

(For simplicity, from here on, we will use d to denote the scale dimension of scalar and fermionic unparticles, instead of $d_{\mathcal{U}_f,\mathcal{U}_s}$). If we make change of variable $p' = p + q$ we get

$$\begin{aligned} \Sigma^f_0(p) &= \Sigma^f_0(p' - q) \\ &= (-1)^{3/2-d_f} ((p')^2 - m^2 + q^2 - 2p' \cdot q)^{3/2-d_f} \\ &= (-1)^{3/2-d} ((p')^2 - m^2 + y)^{3/2-d} \end{aligned} \tag{5.59}$$

Applying a Taylor expansion with respect to y we find

$$\Sigma^f_0(p'-q) = \Sigma^f_0(p')\Big(1 - \frac{3/2-d}{p'^2-m^2}(q^2-2q.p') + \frac{(3/2-d_f)(1/2-d)}{(p'^2-m^2)^2}(q^2-2q.p')^2 + \cdots\Big) \tag{5.60}$$

from this expansion we get

$$\begin{aligned} \Sigma^f_0(p+q) - \Sigma^f_0(p) &= -\left(\Sigma^f_0(p'-q) - \Sigma^f_0(p')\right) \\ &= \frac{3/2-d}{p'^2-m^2}(q^2-2q.p') - \frac{(3/2-d)(1/2-d)}{(p'^2-m^2)^2}(q^2-2q.p')^2 + \cdots \end{aligned} \tag{5.61}$$

Taking the first order in the expansion coefficient $y = q^2 - 2q.p'$, the form factor $\Sigma^f_1(p, q)$ becomes

$$\Sigma^f_1(p, q) \approx (-1)^{3/2-d} \frac{3/2-d}{((p+q)^2 - m^2)^{d-1/2}} \tag{5.62}$$

Using this approximation for $\Sigma_1^f(p,q)$ we can derive a similar approximation for $\Sigma_2^f(p,q_1,q_2)$ as follows

$$\begin{aligned}\Sigma_2^f(p,q_1,q_2) &= \frac{\Sigma_1^f(p,q_1+q_2)-\Sigma_1^f(p,q_1)}{(p+q_1+q_2)^2-(p+q_1)^2}\\ &\approx \frac{(-1)^{3/2-d}(3/2-d)}{(p+q_1+q_2)^2-(p+q_1)^2}\left(\frac{1}{((p+q_1+q_2)^2-m^2)^{d-1/2}}-\frac{1}{((p+q_1)^2-m^2)^{d-1/2}}\right)\\ &= (-1)^{5/2-d}(3/2-d)\frac{((p'+q_2)^2-m^2)^{d-1/2}-(p'^2-m^2)^{d-1/2}}{((p'+q_2)^2-(p')^2)((p'+q_2)^2-m^2)^{d-1/2}(p'^2-m^2)^{d-1/2}}\end{aligned} \tag{5.63}$$

where in this case $p'=p+q_1$. now we take a first order approximation for the expression $((p'+q_2)^2-m^2)^{d-1/2}$

$$\begin{aligned}((p'+q_2)^2-m^2)^{d-1/2} &= (p'^2+q_2^2+2p'\cdot q_2-m^2)^{d-1/2}\\ &= (p'^2-m^2)^{d-1/2}\left(1+\frac{q_2^2+2p'\cdot q_2}{p'^2-m^2}\right)^{d-1/2}\\ &\approx (p'^2-m^2)^{d-1/2}\left(1+(d-1/2)\frac{q_2^2+2p'\cdot q_2}{p'^2-m^2}\right)\end{aligned} \tag{5.64}$$

plugging the result (5.64) in eq(5.63) we find finally

$$\Sigma_2^f(p,q_1,q_2) \approx \frac{(-1)^{5/2-d}(3/2-d)(d-1/2)}{((p+q_1)^2-m^2)((p+q_1+q_2)^2-m^2)^{d-1/2}} \tag{5.65}$$

Using these approximations we can proceed with the calculation of the loop integrals represented in Fig (5.1). We note that the second diagram, which involves interaction of two unfermion with two gauge bosons, does not have an equivalent in the SM. Eq (5.56) can be written as

$$\Pi_{(a)}^{ab\mu\nu} = -\mu^{4-D}\int\frac{d^Dp}{(2\pi)^D}\frac{\mathrm{Tr}\left(\Gamma^{a\mu}(p,q,p+q)(\not p+m)\Gamma^{b\nu}(p+q,-q,-p)(\not p+\not q+m)\right)}{(p^2-m^2)^{5/2-d}((p+q)^2-m^2)^{5/2-d}} \tag{5.66}$$

and Eq(5.57) becomes

$$\Pi_{(b)}^{ab\mu\nu} = -\mu^{4-D}\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left(\Gamma^{ab\mu\nu}(p,q,p+q)(\not p+m)\right)(p^2-m^2)^{5/2-d} \tag{5.67}$$

Substituting the expressions from eq(5.62) and (5.65) in the previews two equations we find the following

$$\Pi^{ab\mu\nu}_{(a)} = -\mu^{4-D}\int\frac{d^Dp}{(2\pi)^D}\Big\{N_1^{\mu\nu}\Big(\frac{1}{(p^2-m^2)^{d-1/2}((p+q)^2-m^2)^{5/2-d}}$$
$$+\frac{1}{(p^2-m^2)^{5/2-d}((p+q)^2-m^2)^{d-1/2}}+\frac{2}{(p^2-m^2)((p+q)^2-m^2)}\Big)+N_2^{\mu\nu}(\frac{3}{2}-d)$$
$$\times\Big(\frac{1}{(p^2-m^2)^{d+1/2}((p+q)^2-m^2)^{5/2-d}}+\frac{1}{(p^2-m^2)^2((p+q)^2-m^2)}\Big)+N_3^{\mu\nu}(\frac{3}{2}-d)$$
$$\times\Big(\frac{1}{(p^2-m^2)^{5/2-d}((p+q)^2-m^2)^{d+1/2}}+\frac{1}{(p^2-m^2)^2((p+q)^2-m^2)}\Big)$$
$$+\frac{N_4^{\mu\nu}(\frac{3}{2}-d)^2}{(p^2-m^2)^2((p+q)^2-m^2)^2}\Big\} \tag{5.68}$$

where

$$\begin{aligned}N_1^{\mu\nu} &= \mathrm{Tr}\big(\gamma^\mu(T_1^a+T_2^a\gamma_5)(\not p+m)\gamma^\nu(T_1^b+T_2^b\gamma_5)(\not p+\not q+m)\big)\\ &= 4\left(\eta^{\mu\nu}\left(m^2A-p\cdot qB-p^2B\right)+\left(2p^\mu p^\nu+q^\mu p^\nu+q^\nu p^\mu\right)B\right)\end{aligned} \tag{5.69}$$

with

$$A=\mathrm{Tr}\left(T_1^aT_1^b-T_2^aT_2^b\right),\ B=\mathrm{Tr}\left(T_1^aT_1^b+T_2^aT_2^b\right) \tag{5.70}$$

$$\begin{aligned}N_2^{\mu\nu} &= (2p+q)^\nu\,\mathrm{Tr}\big(\gamma^\mu(T_1^a+T_2^a\gamma_5)(\not p+m)(2\not p+\not q-2m)(T_1^b+T_2^b\gamma_5)(\not p+\not q+m)\big)\\ &= 4\left(q^\mu\left(Bp^2-Cm^2\right)-p^\mu\left(2m^2-2p\dot q-2p^2-q^2\right)B\right)(2p+q)^\nu\end{aligned} \tag{5.71}$$

with

$$C=\mathrm{Tr}\left(T_1^aT_1^b+3T_2^aT_2^b\right) \tag{5.72}$$

$$\begin{aligned}N_3^{\mu\nu} &= (2p+q)^\mu\,\mathrm{Tr}\big((2\not p+\not q-2m)(T_1^a+T_2^a\gamma_5)\gamma^\nu(T_1^b+T_2^b\gamma_5)(\not p+\not q+m)\big)\\ &= (2p+q)^\mu 4\left(q^\nu\left(Bp^2-Am^2\right)-p^\nu\left(2m^2-2p\dot q-2p^2-q^2\right)B\right)\end{aligned} \tag{5.73}$$

and

$$\begin{aligned}N_4^{\mu\nu} &= \mathrm{Tr}\big((2\not p+\not q-2m)(T_1^a+T_2^a\gamma_5)(\not p+m)(2\not p+\not q-2m)(T_1^b+T_2^b\gamma_5)(\not p+\not q+m)\big)\\ &\quad\times(2p+q)^\mu(2p+q)^\nu\\ &= 4B\Bigg(-p^2\left(8m^2-8p\cdot q-3q^2\right)-p\cdot q(8m^2-q^2)+m^2\left(4m^2-q^2\frac{D}{B}\right)\\ &\quad+2(p\cdot q)^2B+4p^4B\Bigg)(2p+q)^\mu(2p+q)^\nu\end{aligned} \tag{5.74}$$

with

$$D = \mathrm{Tr}\left(3T_1^a T_1^b + T_2^a T_2^b\right) \tag{5.75}$$

In the same manner substituting the approximated form factor (5.65) leads to the following expression for the four point polarization function (5.57)

$$\begin{aligned}\Pi^{ab\mu\nu}_{(b)} &= -\mu^{4-D} g_a g_b \int \frac{d^D p}{(2\pi)^D} \frac{\frac{3}{2}-d}{(p^2-m^2)^{5/2-d}} \Bigg\{ N_1' \Bigg(2\eta^{\mu\nu} \frac{1}{(p^2-m^2)^{d-1/2}} \\ &- \frac{(2p-q)^\nu (2p-q)^\mu (d-\frac{1}{2})}{((p-q)^2-m^2)(p^2-m^2)^{d-1/2}} - \frac{(2p+q)^\nu (2p+q)^\mu (d-\frac{1}{2})}{((p+q)^2-m^2)(p^2-m^2)^{d-1/2}} \Bigg) \\ &+ N_2'^\mu \left(\frac{(2p-q)^\nu}{((p+q)^2-m^2)^{d-1/2}} + \frac{(2p-q)^\nu}{(p^2-m^2)^{d-1/2}} \right) + N_3'^\nu \left(\frac{(2p+q)^\mu}{((p-q)^2-m^2)^{d-1/2}} \right. \\ &\left. + \frac{(2p-q)^\mu}{(p^2-m^2)^{d-1/2}} \right) \Bigg\}\end{aligned} \tag{5.76}$$

where

$$\begin{aligned} N_1' &= 2\,\mathrm{Tr}((\not p + m)(\not p - m)(T_1 + T_2\gamma_5)) \\ &= 8\,\mathrm{Tr}(T_1)(p^2 - m^2), \end{aligned} \tag{5.77}$$

$$\begin{aligned} N_2'^\mu &= \mathrm{Tr}((\not p + m)\gamma^\mu (T_1 + T_2\gamma_5)) \\ &= 4\,\mathrm{Tr}(T_1) p^\mu \end{aligned} \tag{5.78}$$

and

$$\begin{aligned} N_3'^\nu &= \mathrm{Tr}((\not p + m)\gamma^\nu (T_1 + T_2\gamma_5)) \\ &= 4\,\mathrm{Tr}(T_1) p^\nu \end{aligned} \tag{5.79}$$

where T_1 is the operator corresponding to the four point vertex function defined in eq(5.55). With tensor manipulation of the p terms in the numerators, we can reduce all integrals in eq(5.68) and (5.76) into ones of the following form

$$I(k,\alpha,\beta) = \int \frac{d^D p}{(2\pi)^D} \frac{(p^2)^k}{(p^2-m^2)^\alpha ((p+q)^2-m^2)^\beta} \tag{5.80}$$

which is found to be (more details in appendix A)

$$I(k,\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} {}_3F_2 \left(\left\{ 1, \alpha+\beta-\frac{D}{2}-k, \alpha, \beta \right\}, \left\{ \frac{\alpha+\beta}{2}, \frac{\alpha+\beta+1}{2} \right\}, \frac{\tau}{4} \right) \tag{5.81}$$

where ${}_3F_2$ is hypergeometric function that converges for $\frac{\tau}{4} = \frac{q^2}{4m^2} \leq 1$, which is consistent with experimental data since q^2, the momentum transfer in electroweak precision experiments, is in the vicinity of M_Z^2 and we will assume that m the CBS is superior to M_Z. We can express the polarization functions (5.56) and (5.57) in terms of hypergeometric functions like the one in (5.81). Since our goal is to calculate the oblique parameters and the beta function of gauge coupling, we concentrate on the physically relevant part of the polarization functions (5.68,5.76), which is the transverse part, the part proportional to the metric tensor $\Pi^{ab\mu\nu}(q^2) = i\eta^{\mu\nu}\Pi^{ab}(q^2) + \dots$. Hence, the polarization function expressed by the sum of diagram (a) and (b) in Fig (5.1) ($\Pi^{ab} = \Pi^{ab}_{(a)} + \Pi^{ab}_{(b)}$) is the following

$$\Pi^{ab}(q^2) = -8i\frac{g_a g_b}{(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}-2}\Bigg\{\Gamma(2-\frac{D}{2})\Big\{F_1 + (\frac{3}{2}-d)BF_2 + (\frac{3}{2}-d)^2 BF_3\Big\} + \Gamma(3-\frac{D}{2})(\frac{3}{2}-d)^2 BF_4\Bigg\} - 4i\frac{g^2}{(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}-2}\frac{\Gamma(2-\frac{D}{2})}{1-\frac{D}{2}}\,\mathrm{Tr}(T_1)F_5 \tag{5.82}$$

where the superscript a and b denote the SM gauge bosons γ, Z, or W and the functions from F_1 to F_5 are the following

$$\begin{aligned} F_1 &= -Bm^2\Bigg[{}_3F_2\left(\left\{2-\frac{D}{2}, d-\frac{1}{2}, \frac{5}{2}-d\right\}, \left\{1, \frac{1}{2}\right\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 2-\frac{D}{2}, \frac{1}{2}, \frac{\tau}{4}\right)\Bigg] \\ &+Am^2\Bigg[{}_3F_2\left(\left\{3-\frac{D}{2}, d-\frac{1}{2}, \frac{5}{2}-d\right\}, \left\{1, \frac{3}{2}\right\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 3-\frac{D}{2}, \frac{3}{2}, \frac{\tau}{4}\right)\Bigg] \\ &+\frac{Bq^2}{\Gamma(4)}\Bigg[\left(d-\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_3F_2\left(\left\{3-\frac{D}{2}, d+\frac{1}{2}, \frac{7}{2}-d\right\}, \left\{2, \frac{5}{2}\right\}, \frac{\tau}{4}\right) \\ &+{}_2F_1\left(2, 3-\frac{D}{2}, \frac{5}{2}, \frac{\tau}{4}\right)\Bigg] \end{aligned} \tag{5.83}$$

,

$$\begin{aligned} F_2 &= \frac{2m^2-q^2}{\Gamma(3)}\Bigg[{}_3F_2\left(\left\{3-\frac{D}{2}, d+\frac{1}{2}, \frac{5}{2}-d\right\}, \left\{2, \frac{3}{2}\right\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 3-\frac{D}{2}, \frac{3}{2}, \frac{\tau}{4}\right)\Bigg] \\ &-m^2\frac{1+\frac{D}{2}}{1-\frac{D}{2}}\Bigg[{}_3F_2\left(\left\{2-\frac{D}{2}, d+\frac{1}{2}, \frac{5}{2}-d\right\}, \left\{2, \frac{3}{2}\right\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 2-\frac{D}{2}, \frac{3}{2}, \frac{\tau}{4}\right)\Bigg] \\ &+\frac{2q^2}{\Gamma(5)}\Bigg[\left(d+\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_3F_2\left(\left\{3-\frac{D}{2}, d+\frac{3}{2}, \frac{7}{2}-d\right\}, \left\{3, \frac{5}{2}\right\}, \frac{\tau}{4}\right) \\ &+2{}_2F_1\left(2, 3-\frac{D}{2}, \frac{5}{2}, \frac{\tau}{4}\right)\Bigg] \end{aligned} \tag{5.84}$$

,

$$F_3 = \left(1+\frac{D}{2}\right)\left[\frac{-8m^2+3q^2+\frac{2q^2}{n}}{\Gamma(4)}{}_2F_1\left(2,3-\frac{D}{2},\frac{5}{2},\frac{\tau}{4}\right) - 4q^2\frac{\frac{16}{n}+8}{\Gamma(6)}{}_2F_1\left(3,3-\frac{D}{2},\frac{7}{2},\frac{\tau}{4}\right)\right]$$
$$-\frac{4}{\Gamma(4)}m^2\frac{\left(2+\frac{D}{2}\right)\left(1+\frac{D}{2}\right)}{1-\frac{D}{2}}{}_2F_1\left(2,2-\frac{D}{2},\frac{5}{2},\frac{\tau}{4}\right) \quad (5.85)$$

,

$$F_4 = -\frac{4m^2-q^2\frac{D}{B}}{\Gamma(4)}{}_2F_1\left(2,4-\frac{D}{2},\frac{5}{2},\frac{\tau}{4}\right) - 4\frac{\left(8q^2-\frac{q^4}{m^2}\right)}{\Gamma(6)}{}_2F_1\left(3,4-\frac{D}{2},\frac{7}{2},\frac{\tau}{4}\right)$$
$$-4q^2\frac{\Gamma(4)^2}{\Gamma(8)}{}_2F_1\left(4,4-\frac{D}{2},\frac{9}{2},\frac{\tau}{4}\right) \quad (5.86)$$

and

$$F_5 = m^2\left[6-8\left(d-1/2\right){}_2F_1\left(1,2-\frac{D}{2},\frac{3}{2},\frac{\tau}{4}\right)+2{}_3F_2\left(\left\{2-\frac{D}{2},d-\frac{1}{2},\frac{5}{2}-d\right\},\left\{1,\frac{3}{2}\right\},\frac{\tau}{4}\right)\right] \quad (5.87)$$

The result (5.82) is not renormalizable. To find the renormalized polarization function $\Pi^{ab}_{ren}(q^2)$, we use the $\overline{MS}$ renormalization scheme. After separating the divergent and the finite parts, we arrive at the renormalized polarization function (see appendix A for details)

$$\Pi^{ab}_{ren}(q^2) = -i\frac{g_a g_b}{2\pi^2}B(F_1'+(\frac{3}{2}-d)^2F_2') - ig^2\frac{\mathrm{Tr}(T_1)(\frac{3}{2}-d)}{4\pi^2}F_3'$$
$$-\ln\left(\frac{\mu^2}{m^2}\right)\left\{\frac{g_a g_b}{2\pi^2}(F_4'+F_5')+g^2\frac{\mathrm{Tr}(T_1)(\frac{3}{2}-d)^2}{4\pi^2}2m^2\right\} \quad (5.88)$$

where

$$F_1' = m^2\left[-2+\frac{1}{3}\tau\left(d-\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_4F_3\left(1,1,d+\frac{1}{2},\frac{7}{2}-d,2,2,\frac{5}{2},\frac{\tau}{4}\right)\right.$$
$$\left.+\frac{1}{3}\tau\,{}_2F_1\left(1,1,\frac{5}{2},\frac{\tau}{4}\right)\right] + m^2\left(\frac{3}{2}-d\right)\left[4+\frac{1}{4}\tau\left(d+\frac{1}{2}\right)\left(\frac{5}{2}-d\right)\right.$$
$$\left.\times\,{}_4F_3\left(1,1,d+\frac{3}{2},\frac{7}{2},2,3,\frac{5}{2},\frac{\tau}{4}\right)+\frac{1}{2}\tau\,{}_2F_1\left(1,1,\frac{5}{2},\frac{\tau}{4}\right)\right] \quad (5.89)$$

,

$$F_2' = \frac{8m^2 - \frac{11}{4}q^2}{\Gamma(4)} {}_2F_1\left(1, 2, \frac{5}{2}, \frac{\tau}{4}\right) + \frac{24}{\Gamma(6)} q^2 {}_2F_1\left(1, 3, \frac{7}{2}, \frac{\tau}{4}\right)$$
$$\frac{4}{\Gamma(4)} m^2 \left(5 + \frac{12}{5}\tau\, {}_3F_2\left(1, 1, 3, 2, \frac{7}{2}, \frac{\tau}{4}\right)\right) - \frac{4m^2 - q^2\frac{D}{B}}{\Gamma(4)} {}_2F_1\left(2, 2, \frac{5}{2}, \frac{\tau}{4}\right)$$
$$-\frac{4}{\Gamma(6)}\left(8q^2 - \frac{q^4}{m^2}\right) {}_2F_1\left(2, 3, \frac{7}{2}, \frac{\tau}{4}\right) - 4q^2 \frac{\Gamma(4)^2}{\Gamma(8)} {}_2F_1\left(2, 4, \frac{9}{2}, \frac{\tau}{4}\right) \quad (5.90)$$

,

$$F_3' = -6m^2 + 8m^2\left(d - \frac{1}{2}\right)\left(1 + \frac{1}{6}\tau\, {}_2F_1\left(1, 1, \frac{5}{2}, \frac{\tau}{4}\right)\right) - 2m^2\left(1 + \frac{1}{6}\tau\right.$$
$$\left.\times\left(d - \frac{1}{2}\right)\left(\frac{5}{2} - d\right) {}_4F_3\left(1, 1, d + \frac{1}{2}, \frac{7}{2} - d, 2, 2, \frac{5}{2}, \frac{\tau}{4}\right)\right) \quad (5.91)$$

,

$$F_4' = 4Bm^2 + Am^2\left({}_3F_2\left(\{1, d - \frac{1}{2}, \frac{5}{2} - d\}, \{1, \frac{3}{2}\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 1, \frac{3}{2}, \frac{\tau}{4}\right)\right)$$
$$+ B\left(\frac{3}{2} - d\right)^2\left(\frac{3}{\Gamma(4)}\left(-8m^2 + \frac{7}{2}q^2\right) {}_2F_1\left(1, 2, \frac{5}{2}, \frac{\tau}{4}\right) - \frac{144}{\Gamma(6)} q^2\right.$$
$$\left.\times\, {}_2F_1\left(1, 3, \frac{7}{2}, \frac{\tau}{4}\right) + \frac{48}{\Gamma(4)} m^2\right) \quad (5.92)$$

and

$$F_5' = B\left(\frac{3}{2} - d\right)\left\{\frac{2m^2 - q^2}{\Gamma(3)}\left({}_3F_2\left(\{1, d - \frac{1}{2}, \frac{5}{2} - d\}, \{1, \frac{3}{2}\}, \frac{\tau}{4}\right) + {}_2F_1\left(1, 1, \frac{3}{2}, \frac{\tau}{4}\right)\right)\right.$$
$$+ \frac{2q^2}{\Gamma(5)}\left(\left(d + \frac{1}{2}\right)\left(\frac{5}{2} - d\right) {}_3F_2\left(\{1, d + \frac{3}{2}, \frac{7}{2} - d\}, \{3, \frac{5}{2}\}, \frac{\tau}{4}\right) + 2\,{}_2F_1\left(1, 2, \frac{5}{2}, \frac{\tau}{4}\right)\right)$$
$$\left. + 6m^2\right\} \quad (5.93)$$

Now that we have found the general expression for the fermionic polarization function, we can express the polarization functions $\Pi^{\gamma Z}$, Π^{ZZ},$\Pi^{\gamma\gamma}$ and Π^{WW} just by replacing the constants A, B, C and D and $\mathrm{Tr}(T_1)$ for each case. Following the definition of these constants from eqs(5.70,5.72,5.75) and relaying on the definition of the corresponding generators (deduced from the vertex functions (5.38,5.42,5.45,5.47,5.49,5.50,5.53) we find the following results(more detail in appendix B):

For $\Pi^{\gamma\gamma}$ we have

$$\begin{aligned}
A = B &= q_u^2 + q_d^2 \\
D &= 3(q_u^2 + q_d^2) \\
g_a = g_b &= \frac{e}{2},\ g^2 = \frac{e^2}{2} \\
\mathrm{Tr}(T_1) &= q_u^2 + q_d^2
\end{aligned} \tag{5.94}$$

For Π^{ZZ} we have the following

$$\begin{aligned}
A &= 2\sin^2(\theta_W)(q_d - q_u + 2\sin^2(\theta_W)(q_u^2 + q_d^2)) \\
B &= 1 + 2\sin^2(\theta_W)(q_d - q_u) + 4\sin^4(\theta_W)(q_u^2 + q_d^2) \\
D &= 2 + 6\sin^2(\theta_W)(q_d - q_u) + 12\sin^4(\theta_W)(q_u^2 + q_d^2) \\
g_a = g_b &= \frac{e}{4\sin(\theta_W)\cos(\theta_W)},\ g^2 = \frac{e^2}{4\sin^2(\theta_W)\cos^2(\theta_W)} \\
\mathrm{Tr}(T_1) &= \frac{1}{2}(1 + 2\sin^2(\theta_W)(q_d - q_u) + 4\sin^4(\theta_W)(q_u^2 + q_d^2)),
\end{aligned} \tag{5.95}$$

For $\Pi^{\gamma Z}$ we get

$$\begin{aligned}
A = B &= \frac{1}{2}(q_u - q_d - 4\sin^2(\theta_W)(q_u^2 + q_d^2)) \\
D &= \frac{3}{2}(q_u - q_d - 4\sin^2(\theta_W)(q_u^2 + q_d^2)) \\
g_a &= \frac{e}{2},\ g_b = \frac{e}{4\sin(\theta_W)\cos(\theta_W)},\ g^2 = \frac{e^2}{8\sin(\theta_W)\cos(\theta_W)} \\
\mathrm{Tr}(T_1) &= \frac{1}{2}(q_u - q_d),
\end{aligned} \tag{5.96}$$

and for Π^{WW} we have

$$\begin{aligned}
A &= 0 \\
B &= 1 \\
D &= 2 \\
g_a = g_b &= \frac{e}{4\sin(\theta_W)},\ g^2 = \frac{e^2}{8\sin^2(\theta_W)} \\
\mathrm{Tr}(T_1) &= \frac{1}{2},
\end{aligned} \tag{5.97}$$

with the computation of the self energy functions to the electroweak gauge bosons induced by unfermions, we can proceed with the calculation of the oblique parameters

5.3 Oblique parameters for unfermions

As we have seen in chapter 2, the oblique parameters are used to represent the contribution of new physics models to electroweak observable measured at low energy. In this work, we will use this approach to constrain the parameters space of the gauged unparticle model described previously. In the literature there are six independent parameters . The STU parameters were introduced in the original paper of Peskin and Takachi [18]. In that paper, the authors made the assumption that the new physics is much heavier than the masses of the gauge bosons M_Z, M_W which allowed them to consider a linear approximation to calculate the polarization functions Π^{ab}. In our case, the equivalent to the mass of the new states is the CSB scale m. We de not known at what energy scale the conformal symmetry is broken, but we will assume m is in the range $[100, 1000]$ Gev. We begin by defining the S and T parameters in terms of the polarization functions Π^{ab} as follows

$$S = \frac{4s_w^2 c_w^2}{\alpha}\left(\frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} - \frac{c_w^2 - s_w^2}{s_w c_w}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right) \tag{5.98}$$

$$T = \frac{1}{\alpha}\left(\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right) \tag{5.99}$$

where the derivatives $\Pi'_{ab}(q^2)$ are defined by $\Pi'_{ab}(q^2) = d\Pi_{ab}(q^2)/dq^2$. α is the fine structure constant and $s_w = \sin(\theta_W)$, $c_w = \cos(\theta_W)$. Using the results for the polarization functions (5.88) and making the appropriate substitution from eqs(5.94…5.97)) in (5.98) and (5.99) we find the expressions for the oblique parameters to be

$$\begin{aligned} T = \frac{m^2}{8\pi s_w^2 c_w^2 M_Z^2}\Bigg\{ & \frac{23}{6} + 4d - 2d^2 + \left(1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)\right)\Bigg(-\frac{7}{2} + 6d - \frac{10}{3}d^2 \\ & -\frac{2}{3}\left(\frac{3}{2} - d\right)^2 \frac{1 + 3s_w^2(q_d - q_u) + 6s_w^4(q_u^2 + q_d^2)}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)}\Bigg) + \ln\left(\frac{\mu^2}{m^2}\right)\Bigg(-\frac{5}{4} + \frac{14}{2}d - 4d^2 \\ & + \left(1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)\right)\Bigg(43 - 44d + 12d^2 \\ & + \frac{4s_w^2\left((q_d - q_u) + 2s_w^2(q_u^2 + q_d^2)\right)}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2}\Bigg)\Bigg)\Bigg\} \end{aligned} \tag{5.100}$$

and

$$S = -\frac{(1+2s_w^2(q_d-q_u)+4s_w^4(q_u^2+q_d^2))}{2\pi}\left(F_1''+F_2''+\ln\left(\frac{\mu^2}{m^2}\right)(F_3''+F_4''+F_5'')\right)$$
$$+\frac{c_w^2-s_w^2}{48\pi}\left(\frac{q_d-q_u+2s_w^2(q_u^2+q_d^2)}{70}\left(6115-6219d+568d^2+420d^3\right)+8(q_d-q_u)\right.$$
$$\left.\times(3/2-d)^2(d-1/2)+\ln\left(\frac{\mu^2}{m^2}\right)\left(q_d-q_u-4s_w^2(q_u^2+q_d^2)\right)\left(-16-33d+28d^2-4d^3\right)\right)$$
$$+\frac{s_w^2c_w^2(q_u^2+q_d^2)}{24\pi}\left(\frac{7123-4959d+848d^2-140d^3}{70}+\ln\left(\frac{\mu^2}{m^2}\right)\left(-16-33d+28d^2-4d^3\right)\right) \tag{5.101}$$

where

$$F_1'' = \frac{1}{3}\left(d-\frac{1}{2}\right)^2\left(\frac{5}{2}-d\right){}_4F_3\left(1,1,d+\frac{1}{2},\frac{7}{2},2,2,\frac{5}{2},\frac{\tau_1}{4}\right)+\frac{1}{4}\left(d+\frac{1}{2}\right)$$
$$\times\left(\frac{5}{2}-d\right)\left(\frac{3}{2}-d\right){}_4F_3\left(1,1,d+\frac{3}{2},\frac{7}{2}-d,2,3,\frac{5}{2},\frac{\tau_1}{4}\right)+\left(\frac{3}{2}-d\right)^2$$
$$\times\left(\left(\frac{4}{3}\frac{m^2}{M_Z^2}-\frac{11}{24}\right){}_2F_1\left(1,2,\frac{5}{2},\frac{\tau_1}{4}\right)-\frac{4}{3}\frac{m^2}{M_Z^2}+\frac{1}{5}\left({}_2F_1\left(1,3,\frac{7}{2},\frac{\tau}{4}\right)\right.\right.$$
$$\left.\left.+8{}_3F_2\left(1,1,3,2,\frac{7}{2},\frac{\tau_1}{4}\right)\right)\right) \tag{5.102}$$

$$F_2'' = -\left(\frac{3}{2}-d\right)^2\left(\left(\frac{2}{3}\frac{m^2}{M_Z^2}-\frac{1}{3}\right)\frac{1+3s_w^2(q_d-q_u)+6s_w^4(q_u^2+q_d^2)}{1+2s_w^2(q_d-q_u)+4s_w^4(q_u^2+q_d^2)}{}_2F_1\left(2,2,\frac{5}{2},\frac{\tau_1}{4}\right)\right.$$
$$+\frac{1}{30}(8-\tau_1){}_2F_1\left(2,3,\frac{7}{2},\frac{\tau_1}{4}\right)+\frac{1}{35}{}_2F_1\left(2,4,\frac{9}{2},\frac{\tau_1}{4}\right)-\frac{2m^2}{3M_Z^2}$$
$$\left.\times\frac{1+3s_w^2(q_d-q_u)+6s_w^4(q_u^2+q_d^2)}{1+2s_w^2(q_d-q_u)+4s_w^4(q_u^2+q_d^2)}\right)+\left(\frac{1}{12}+\frac{13}{6}d-\frac{4}{3}d^2\right){}_2F_1\left(1,1,\frac{5}{2},\frac{\tau_1}{4}\right) \tag{5.103}$$

and

$$F_3'' = \frac{1}{6}\left(\left(d-\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_3F_2\left(1,d+\frac{1}{2},\frac{7}{2}-d,2,\frac{5}{2},\frac{\tau_1}{4}\right)+{}_2F_1\left(1,2,\frac{5}{2},\frac{\tau_1}{4}\right)\right)$$
$$+\left(\frac{3}{2}-d\right)^2\left(\left(-4\frac{m^2}{M_Z^2}+\frac{\tau_1}{4}\right){}_2F_1\left(1,2,\frac{5}{2},\frac{\tau_1}{4}\right)-\frac{6}{5}{}_2F_1\left(1,3,\frac{7}{2},\frac{\tau_1}{4}\right)+4\frac{m^2}{M_Z^2}\right) \tag{5.104}$$

and

$$F_4'' = \left(\frac{3}{2} - d\right)\left(\left(\frac{m^2}{M_Z^2} - \frac{1}{2}\right)\left({}_3F_2\left(1, d+\frac{1}{2}, \frac{5}{2} - d, 2, \frac{3}{2}, \frac{\tau_1}{4}\right) + {}_2F_1\left(1, 1, \frac{3}{2}, \frac{\tau_1}{4}\right)\right)\right.$$
$$-2\frac{m^2}{M_Z^2} + \frac{2}{\Gamma(5)}\left(\left(d+\frac{1}{2}\right)\left(\frac{5}{2} - d\right){}_3F_2\left(1, d+\frac{3}{2}, \frac{7}{2} - d, 3, \frac{5}{2}, \frac{\tau_1}{4}\right)\right.$$
$$\left.\left. +2{}_2F_1\left(1, 2, \frac{5}{2}, \frac{\tau_1}{4}\right)\right)\right) \quad (5.105)$$

and

$$F_5'' = \frac{2s_w^2\,(q_d - q_u + 2s_w^2(q_u^2 + q_d^2))}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)}\left({}_3F_2\left(1, d-\frac{1}{2}, \frac{5}{2} - d, 1, \frac{3}{2}, \frac{\tau_1}{4}\right)\right.$$
$$\left. + {}_2F_1\left(1, 1, \frac{3}{2}, \frac{\tau_1}{4}\right) - 2\right) \quad (5.106)$$

where $\tau_1 = \frac{M_Z^2}{m^2}$

5.3.1 Phenomenology

To find the region of parameter space of fermionic unparticle that is compatible with experimental limits, we must compare the unparticle contribution to the oblique parameters S and T to their experimental values deduced from electroweak precision measurements. Taking into account the discovery of the Higgs boson with mass $m_h = 125.18 \pm 0.16$, the fitted values of S and T, as reported in ref [20], are the followings

$$\begin{aligned} \Delta S &= S - S_{SM} = 0.05 \pm 0.11 \\ \Delta T &= T - T_{SM} = 0.09 \pm 0.13 \end{aligned} \quad (5.107)$$

To illustrate the bounds on unparticles parameters from electroweak precision tests we present in Fig. 5.2 and Fig. 5.3 contour plots in the plane of (d, m) in the regions $1,5 \leq d \leq 2.5$ and $100 \leq m \leq 1100$ for S and $100 \leq m \leq 250$ for T. In this study, we have chosen the values $q_u = -1$, $q_d = 0$ for the charges of the upper and lower components, respectively, of the unparticle multiplet. In Fig. 5.2 contour plots for experimental upper and lower bounds $S = 0.11$ and $S = -0.11$ are depicted for two choices of the renormalization scale μ. For $\mu = M_Z/2$, the solid line in the right hand side represents $S = 0.11$ and the solid line in the left hand side represents $S = -0.11$. For $\mu = 2M_Z$, the dashed line in the right represents $S = 0.11$ and the dashed line in the left represents $S = -0.11$. As can bee seen from this figure, for values of scale dimension $d \leq 1,7$, there is practically no constraints on the values of conformal breaking scale m, but for values of $d \geq 1.7$ m is restricted to $m \leq 200\,Gev$. The allowed region in the parameters space (d, m) becomes narrower as d increases. For $\mu = 2M_Z$ the scale dimension d must be inferior to 1.7 to satisfies the experimental bounds. Fig. 5.3 shows contour plots

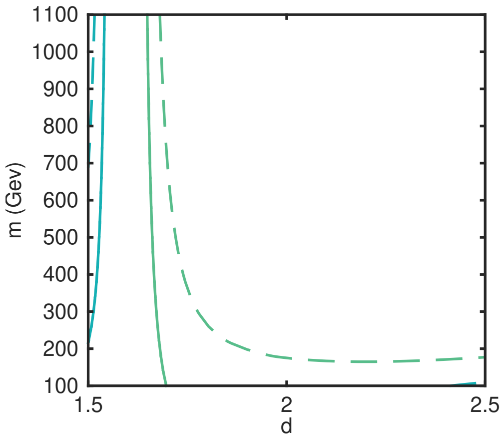

Figure 5.2: contour plots in the plane (d,m) for $S = 0.11$ on the right hand side and $S = -0.11$ on the left hand side, solid lines are contour plots for $\mu = 2M_Z$ and dashed lines are contour plots for $\mu = M_Z/2$.

for the upper and lower experimental limits $T = 0.13$(the upper solid and dashed lines) and $T = -0.13$(the lower solid and dashed lines). The solid plots represent T for the renormalization scale value $\mu = M_Z$ and the dashed plots represent T for $\mu = 2M_Z$. The region between the two solid lines and the two dashed lines is consistent with measurements for the chosen renormalization scale value. It is clear from this figure that the oblique parameter T imposes a strong constraint on the allowed region of parameter space. For $\mu = 2M_Z$, values of the conformal breaking scale $m \geq 200$ are excluded in the range $1.5 \leq d \leq 2.5$. For $\mu = M_Z$, the allowed region is even smaller. The allowed values of the scale dimension d shrinks to the range $1.5 \leq d \leq 1.7$ and $m \leq 110$.

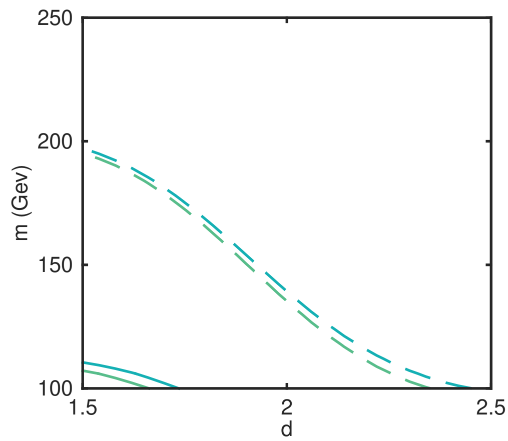

Figure 5.3: contour plots in the plane (d,m) for $T = 0,13$ represented by the upper solid and dashed lines and $T = -0,13$ represented by the lower solid and dashed lines, solid lines are contour plots for $\mu = M_Z$ and dashed lines are for $\mu = 2M_Z$.

Fig. 5.2 and Fig. 5.3 are based on the bounds expressed by Eq. (5.107), in which S and T are taken as independent parameters. In reality, there is a correlation between these two observables expressed by the correlation coefficient $\rho = 0.9$ [20]. Fig. 5.4 shows scatter plots in the (d, m) plane compatible with 1σ experimental bounds of electroweak precision data, in which the correlation coefficient ρ is taken into account. The blue dots represent scatter points for the renormalization scale value $2M_Z$. The red point represents the allowed region for $\mu = M_Z$. From this figure, we see that the allowed region is highly sensitive to the value of the renormalization scale in the chosen range. The IR cut-off scale m is constrained to the interval $100 \leq m \leq 200$, but the scale dimension can take value up to 2.34 for $\mu = 2M_Z$. In general, the combined fitted results of S and T, expressed by Fig. 5.4, are compatible with the restrictions imposed by the oblique parameter T (Fig. 5.3) except that the allowed region gets smaller in the edges, when d approaches 2.4 and the conformal breaking scale m approaches 200.

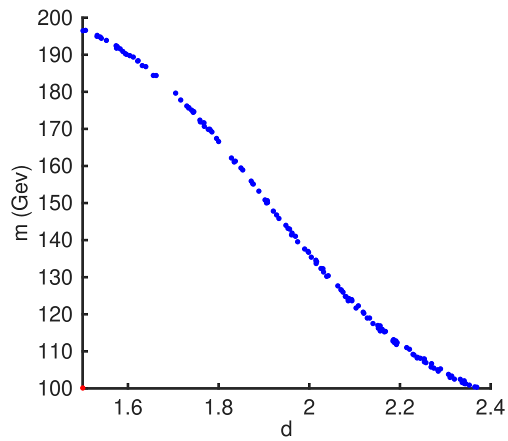

Figure 5.4: scatter plot in the plane (d, m) which show the region in parameters space compatible with the 1σ experimental bound.

5.4 Oblique parameters for scalar unparticles

Following similar steps as in the fermionic unparticle case, in this section we compute the scalar unparticle contribution to oblique parameters, then we use the results to find bounds on the parameters space of the scalar unparticle sector.

the couplings between SM gauge bosons and scalar unparticle are expressed by the vertex functions given in eqs(5.11,5.13). The scalar unparticle contribution to the polarization functions Π^{ab} is depicted in Fig (5.5). The right hand side diagram (a) is

$$\Pi^{ab\mu\nu}_{s(a)} = \mu^{4-D} \int \frac{d^D p}{(2\pi)^D} \operatorname{Tr}\left(\Gamma^{a\mu}(p, q, p+q) S(p) \Gamma^{b\mu}(p+q, -q, -p) S(p+q)\right) \tag{5.108}$$

and diagram (b) is

$$\Pi^{ab\mu\nu}_{s(b)} = \mu^{4-D} \int \frac{d^D p}{(2\pi)^D} \operatorname{Tr}\left(\Gamma^{ab\mu\nu}(p, q, -q) S(p)\right) \tag{5.109}$$

where $S(p)$ is the scalar propagator given by

$$S(p) = \frac{A(d)}{2\sin(\pi d)} \frac{i}{(m^2 - p^2 - i\epsilon)^{2-d}} \tag{5.110}$$

The structure of Feynman vertices $\Gamma^{a\mu}$, $\Gamma^{ab\mu\nu}$ and the propagator $S(p)$ makes the calculation of the integrals (5.108,5.109) complicated. To deal with

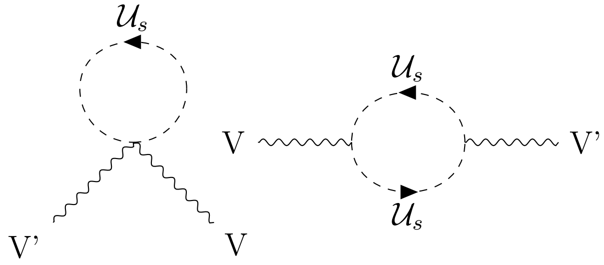

Figure 5.5: The one loop contribution to polarization functions from charged scalar unparticle fields, V and V' stand for γ, Z or W.

this problem, we use a Taylor expansion of the vertices (5.11,5.13), as we did for the unfermion case. For the form factors Σ_1^s and Σ_2^s (Eqs(5.14,5.15)) we get the following results

$$\Sigma_1^s(p,q) \approx (-1)^{2-d} \frac{2-d}{((p+q)^2-m^2)^{d-1}} \tag{5.111}$$

,

$$\Sigma_2^s(p,q_1,q_2) \approx (-1)^{2-d} \frac{(2-d)(1-d)}{((p+q_1+q_2)^2-m^2)^{d-1}((p+q_1)^2-m^2)} \tag{5.112}$$

the asymptotic forms for the vertices (5.11) and (5.13) are the following

$$\Gamma^{a\mu}(p,q,p+q) \simeq ig_a T^a \frac{2\sin(\pi d)}{A(d)} (2p+q)^\mu (-1)^{2-d} \frac{2-d}{((p+q)^2-m^2)^{d-1}} \tag{5.113}$$

$$\begin{aligned} ig^2\Gamma^{ab\mu\nu}(p,q_1,q_2) \simeq ig^2 \frac{2\sin(\pi d)}{A(d)} (-1)^{2-d} \frac{2-d}{((p+q_1+q_2)^2-m^2)^{d-1}} \Big(\eta^{\mu\nu}\{T^a,T^b\} \\ + T^aT^b \frac{(1-d)(2p+q_2)^\nu(2p+2q_2+q_1)^\mu}{(p+q_2)^2-m^2} \\ + T^bT^a \frac{(1-d)(2p+q_1)^\mu(2p+2q_1+q_2)^\nu}{(p+q_1)^2-m^2} \Big) \end{aligned}$$

,

Using these approximations, the first diagram (a) is

$$\Pi^{ab\mu\nu}_{s(a)} = \mu^{4-D} g_a g_b \,\mathrm{Tr}\left(T^aT^b\right)(2-d)^2 \int \frac{d^Dp}{(2\pi)^D} \frac{(2p+q)^\mu(2p+q)^\nu}{(p^2-m^2)((p+q)^2-m^2)} \tag{5.114}$$

and the second diagram is the following

$$\Pi^{ab\mu\nu}_{s(b)} = -\mu^{4-D} g_a g_b \operatorname{Tr}\left(T^a T^b\right)(2-d) \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 - m^2} \times \left(2\eta^{\mu\nu} + (1-d)\frac{(2p-q)^\mu (2p-q)^\nu}{(p-q)^2 - m^2} + (1-d)\frac{(2p+q)^\mu (2p+q)^\nu}{(p+q)^2 - m^2}\right) \tag{5.115}$$

Thus, we have

$$\Pi^{ab\mu\nu}_{s} = \Pi^{ab\mu\nu}_{s(a)} + \Pi^{ab\mu\nu}_{s(b)} = \mu^{4-D}(2-d) g_a g_b \operatorname{Tr}\left(T^a T^b\right) \int \frac{d^D p}{(2\pi)^D} \left(\frac{-2\eta^{\mu\nu}}{p^2 - m^2} - (1-d) \times \frac{(2p-q)^\mu (2p-q)^\nu}{((p-q)^2 - m^2)(p^2 - m^2)} + \frac{(2p+q)^\mu (2p+q)^\nu}{(p^2 - m^2)((p+q)^2 - m^2)}\right) \tag{5.116}$$

To perform the loop integrals in (5.116), we proceed in the same manner as we did in the fermionic case explained before. Here we just give the final result which is the following

$$\Pi^{ab\mu\nu}_{s} = i\frac{(2-d) g_a g_b \operatorname{Tr}\left(T^a T^b\right) m^2}{8\pi^2} \eta^{\mu\nu} \left(1 + \ln\left(\frac{\mu^2}{m^2}\right) + d g(\tau) - d(1 + \ln\left(\frac{\mu^2}{m^2}\right)) f(\tau)\right) \tag{5.117}$$

where

$$f(\tau) = 1 - \frac{\tau}{6}$$
$$g(\tau) = -\frac{\tau}{\Gamma(4)} {}_2F_1\left(1, 1, \frac{5}{2}, \frac{\tau}{4}\right) + \tau^2 \frac{\Gamma(3)^2}{\Gamma(6)} {}_3F_2\left(1, 1, 3, 2, \frac{7}{2}, \frac{\tau}{4}\right)$$

with $\tau = \frac{q^2}{m^2}$ Now, to find the polarization functions $\Pi^{\gamma\gamma}, \cdots$ we must replace g_a, g_b and T^a, T^b defined by eqs(5.38…5.53) in section (5.2) in eq(5.117). Finnaly, the scalar unparticle contribution to the oblique parameters S and T are the following

$$S = \frac{s_w^2 c_w^2 d(2-d)}{\pi} \left\{ \frac{\frac{1}{2} + 4 s_w^2 (q_u^2 + q_d^2) + 2 s_w^2 (q_d - q_u)}{2 s_w^2 c_w^2} \times \left(-\frac{1}{6} + \frac{1}{\Gamma(4)} {}_2F_1\left(1, 1, \frac{5}{2}, \frac{\tau_1}{4}\right) + \tau_1 \frac{\Gamma(3)^2}{\Gamma(6)} {}_3F_2\left(1, 1, 3, 2, \frac{7}{2}, \frac{\tau_1}{4}\right) - \ln\left(\frac{\mu^2}{m^2}\right)\right) + \frac{c_w^2 - s_w^2}{s_w^2 c_w^2} \frac{\frac{1}{2}(q_d - q_u) - 2 s_w^2 (q_u^2 + q_d^2)}{6} \ln\left(\frac{\mu^2}{m^2}\right) + \frac{q_u^2 + q_d^2}{3} \ln\left(\frac{\mu^2}{m^2}\right) \right\} \tag{5.118}$$

and

$$T = \frac{(2-d)(1+d)}{4\pi s_w^2 c_w^2} \frac{m^2}{M_Z^2} \left(1 + \ln\left(\frac{\mu^2}{m^2}\right)\right) \left(\frac{3}{4} - 2s_w^4(q_u^2 + q_d^2) - s_w^2(q_d - q_u)\right) \tag{5.119}$$

5.4.1 Phenomenology

In order to find the region of parameters space of the scalar unparticle model compatible with experiments, we should compare the contribution of scalar unparticle to the oblique parameters S and T to their experimental limits expressed by equation (5.107). In this phenomenological study, we focus on

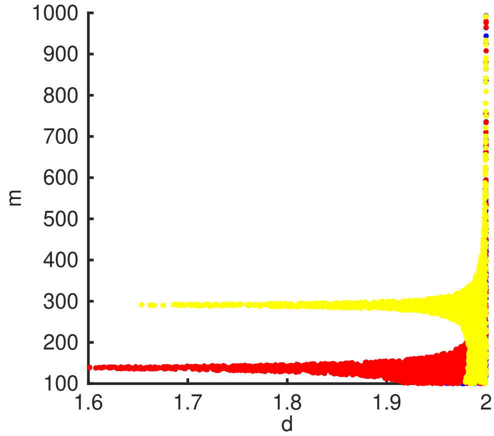

Figure 5.6: scatter plot in the plane (d, m) which shows the region in parameters space compatible with the 1σ experimental bound.

the scatter plot Fig (5.6) which expresses the 1σ electroweak bound from both S and T, taking into account the correlation between the two observables. The red dots represent the scatter plot for the renormalization scale value $\mu = M_Z$. The yellow dots represent scatter points for $\mu = 2M_Z$. From this figure, we see that there is no constraint on the value of the CBS m toward the decoupling limit $d \to 2$. For $\mu = M_Z$ there is a thin region in the parameters space which is compatible with experimental bounds in the range $1 < d < 1.8$, between $1.8 < d < 2$ the constraints on m become more relaxed and it can takes values up to 300 Gev when $d \to 2$.
For $\mu = 2M_Z$, we note from the figure that the range $1 < d < 1.65$ lie outside the experimental bound, but as d approach $d = 1.9$, the compatible region becomes larger.

In conclusion, we have shown, in this study, that the oblique parameters impose a strong constraint on the CBS(m) value. Generally speaking, this value must be inferior to 200 *Gev* to be compatible with electroweak precision measurements. We note that constraints imposed by the OP on the value of the scale dimension d of scalar unparticles are stronger then that off fermionic unparticle. m is a parameter that represent a cut-off under which unparticle degree of freedom disappear. an upper limit of 200 means that the breaking of scale invariance must take place in the energy range inferior to 200 in order to be compatible with experiments. For scalar unparticles this constraints becomes more relaxed as we increase the value of scale dimension d. In the next section, we give remarks concerning unparticle phenomenology in general and its relation to our work with oblique parameters.

5.5 Comments on unparticle phenomenology

A lot of phenomenological studies prove that unparticles can decay, just like normal particles. They can be regarded as a sum over several particle propagators, where the particles have a continuously distributed mass and a width related to the imaginary part of the loop correction as required by unitarity. Scalar unparticles with these interactions can be produced at colliders through gluon-gluon fusion, in the subprocesses $gg \to U$,$gg \to gU$.The unparticle can decay through the processes $U \to gg$ and $U \to \gamma\gamma$, leading to multijet events, or events with two photons plus jets.For the scale dimension $d = 1.1$ and $d = 1.4$ [50]. For larger values of the scale breaking m the decays are almost all prompt. For small m, more unparticles with a long lifetime can be produced, and we get a large number of monojets. This provides a new type of signal of unparticles. Note that if the unparticle is a singlet under SM gauge group transformations, there is a limited number of ways that the unparticles can couple to SM particles [51] . Another scenario is when the unparticles have electroweak quantum numbers. For example unquarks can decay into ordinary quarks and will have a resemblance to a 4th family. It is very important to mention that unparticles can decay even if they are singlets under SM gauge group transformationss (they do not carry SM quantum numbers) [52]. If we consider unparticles as a 4th generation quarks, the collider bounds on masses, precision observables and the renormalization flow of coupling are equivalent to imposing constraints on gauged unparticles parameters which depend on the process and type of unparticles (scalar, vector, spinor or tensor) under consideration. The analysis of electroweak precision tests imposes severe bounds on the involved parameter space, particularly the quark mixing between the third and 4th family and the possible mass differences within the new quark and lepton doublets Constraints on the masses of the 4th family fermions are obtained from their contributions to the electroweak correction parameters S and T. The CCollaboration put a lower bound on the mass of 4th generation up-type quark of about 450 GeV and exclude 4th generation down-type quark in the mass region $255 - 361 GeV$ at 95% C.L [53].

To see the effect of the experimental results on the unparticles parameters space when decaying or interacting with SM particles let us consider the work of ref.[54] where it was shown that the scalar and spinor colored unparticle loop contributions have an important impact on Higgs phenomenology at the LHC and can explain the excess in $h \to \gamma\gamma$ observed by ATLAS experiment. In fact in the scalar case, an enhancement in the Higgs diphoton decay rate requires a negative coupling $\lambda h U_s$ and a large electrical charge to restore the naturalness and vacuum stability, while in the spinor case an enhancement can be obtained by either negative or positive coupling to the Higgs boson depending on the scale dimension d_f due to the flipping of the sign of the spin-1/2 contributions. In both cases, a significant enhancement of $h \to \gamma\gamma$ selects a very special region of the unparticle parameters.The present data of ATLAS in diphoton decay rate of SM-like Higgs boson around $125 GeV$ serve to constrain the unparticle parameter model. Concerning the uncolored unparticles, both scalar and tensorial interactions to SM fields can lead to sizable observable effects in the invariant mass distributions of dilepton pairs at hadron colliders in the large invariant mass region [55, 56]. energy from the unparticle at hadronic collisions are explored. The complex phase in the unparticle propagator that can give rise to interesting interference effects between an unparticle exchange diagram and the standard model amplitudes are found sensitively depending not only on the scale dimension but also on the spin of the unparticle. Furthermore the possible effects of unparticules through photon-photon scattering, rare annihilation type B decays, top quark rare decays and comparaison with experimental data put severe constraints on the unparticles parameter space. As a concrete exemple for the triangle exchange of fermionic unparticles to saturate the upper bound of the electron, muon and neutron electromagnetic dipole moments, one has to have $\Lambda_U = 1 TeV$(Energy scale at which unparticles emerge), $m = 200 GeV$, $d \in [1.5, 2]$, [57]. In the electroweak gauge boson W scattering and since the vector unparticles propagator dependes on the scale dimension d measuring the angular distribution of the W boson, one can determine the scale dimension d. For the scalar signal at the LHC [58], A detailed study of certain processes within the unparticles scenario $pp \to 4\gamma \ldots \; pp \to 2\gamma 2g \ldots pp \to 2\gamma 2l$, $pp \to 4e$ ... $pp \to 4\mu \ldots \; pp \to 2e2\mu$ at $\sqrt{s} = 14 TeV$ is carried out. Using basic selection cuts and analyse various distributions to discriminate the signals over the SM background. Using the experimental values, limits on the uncolored scalar unparticles parameters d and Λ_U are set. The bound on Λ_U can get as large as $1TeV$ for small d values, but it is smaller for larger values. Finally, we conclude that. the unparticles (gauged or ungauged) decays and interactions with the SM particles are very important and lead to sizeable effects. Imposing the collider and/or electroweak precision data tests which are in general complementary will affect more the parameter space region like (Λ_U, d) etc... depending on the type of unparticles (scalars, vectors....) and the process under consideration. In the present case if we consider the unparticles effect like the one of the 4th generation of quarks, we believe that the collider bounds on the 4th generation of fermions will impose stringent constraints on the other gauged fermionic unparticles parameters like the unparticle SM charge Q_U and Yukawa coupling λ_U, and Λ_U.

5.6 Effects of unparticle on gauge coupling

In this section, we study the effects of scalar unparticle on the running of the SM gauge couplings and the unification scale. For this purpose, we consider unparticle contribution, at the one loop level, to the beta function of the SM gauge groups $U(1)_Y$, $SU(2)$ and $SU(3)_c$. In previews works, the authors of [59],[60] have only considered unparticles defined in the r representation of the color group $SU(3)_c$, but neutrals under the weak and hypercharge groups $SU(2)$ and $U(1)_Y$. In our work, we take unparticles charged under all three gauge groups and compute their contribution to the beta function. Then, we search for the unification scale by scanning the parameter space of scalar unparticles which consist of the scale dimension d, and the number of species n_s. We have found that the unification of the three gauge couplings takes place for a large number of unparticle species, $n_s = 9$, for the scale dimension value $d = 1.5$ and the corresponding unification scale is $M_U = 10^{12}$, which is orders of magnitude lower than the grand unification scale (GUS) achieved using supersymmetric particles [61]. We note that in our work, we did not use a canonical normalization for the hyper-charge group coupling g as is usualy done in grand unification models [62].

5.6.1 The beta function

The coupling constants are defined as effective values at some energy scale. This is a characteristic of QFT discovered by Wilson, Polchenski and others. Dynamical constants such as mass, coupling strength are constants in a particular energy range but change value when we go from low energy to high energy. these variations are governed by the renormalization group equation (RGE). For our purpose, we focus on RGE for gauge coupling. The one loop RGE for the SM is given by

$$\mu\frac{dg_i}{d\mu} = \frac{b_i}{4\pi}g_i^3 = \beta_i(g_i),\ i = 1\dots N \tag{5.120}$$

where N is the number of gauge coupling in the theory. μ is the energy scale at which g_i is evaluated and β_i is the beta function for the gauge coupling g_i. In the SM, the gauge couplings are g_S, g and g' corresponding to the gauge groups $SU(3)_c$, $SU(2)$ and $U(1)_Y$ respectively. The constants b_i encode the contribution, at one loop level, of virtual particles to self energy diagrams of the SM gauge particles. Any contribution from new physics, in the form of extra degree of freedom circulating in the loops, can be added to the constants b_i. Here, we consider the contribution of scalar unparticles and in this case, b_i can be written as

$$b_i = b_i^{SM} + b_i^{\mathcal{U}} \tag{5.121}$$

where b_i^{SM} is the contribution of the SM particles given by

$$\begin{aligned} b_1 &= -7 \\ b_2 &= -\frac{19}{6} \\ b_3 &= \frac{41}{10} \end{aligned} \tag{5.122}$$

and $b_i^{\mathcal{U}}$ is the contribution of the unparticle sector. In section (5.4) we already calculated scalar unparticle contribution to the polarization functions of the SM gauge bosons. That result will be used to compute the unparticle beta function for each SM gauge coupling.

The beta function β is related to the polarization functions of SM gauge bosons via the counter terms δ_3 used to eliminate the divergent part of these functions. we can write β as follows

$$\beta = g\mu\frac{\partial}{\partial\mu}\left(\frac{g}{2}\delta_3\right) \tag{5.123}$$

Since δ_3 is proportional to the divergent part of the polarization functions Π^{ab}, here we give its expression in the scalar unparticle case extracted from equation (5.116)

$$\Pi^{ab}_{div} = -\mu^{4-D} 2i \frac{g_a g_b}{(4\pi)^{\frac{D}{4}}} (m^2)^{\frac{D}{2}-1} \operatorname{Tr}\big(T^a T^b\big)(2-d) n_s$$
$$\times\, \Gamma(\frac{D}{2}-1)\left(1 + d\int_0^1 dx \left(1 - \frac{q^2}{m^2} x(1-x)\right)^{\frac{D}{2}-1}\right) \qquad (5.124)$$

in this equation we added the number of unparticle species n_s. The next step is to rewrite eq (5.124) to be proportional to $\Gamma(\epsilon) = \Gamma(2 - \frac{D}{2})$. Using the identity

$$\Gamma(1 - \frac{D}{2}) = \frac{\Gamma(2 - \frac{D}{2})}{1 - \frac{D}{2}}$$

and a Taylor expansion to the first order in ϵ, we find the expression for δ_3

$$\delta_3 = \frac{g_a g_b}{((4\pi)^2} (m^2)^{\frac{D}{2}-2} d(2-d) \operatorname{Tr}\big(T^a T^b\big) \times -\frac{1}{3} n_s \qquad (5.125)$$

substituting in eq(5.123), we find the one loop beta function for scalar unparticle

$$\beta(g) = \frac{1}{48(\pi)^2} g^3 d(2-d) C(r) n_s \qquad (5.126)$$

where $C(r) = \operatorname{Tr}\big(T^a T^b\big)$ is the quadratic Casimir operator for the representation r of the gauge group. From the last equation, we can write the unparticle contribution to the running coupling, $b_i^{\mathcal{U}}$, as follows

$$b_i^{\mathcal{U}} = \frac{1}{12\pi} g^3 d(2-d) C(r) n_s \qquad (5.127)$$

To calculate the unparticle contribution to the running of gauge coupling, we need to add $b_i^{\mathcal{U}}$ to the coefficients b_i of the SM. However, since unparticle fields have not been detected at low energy experiments, scale invariance must be broken at some energy scale m. So, unparticle fields do not contribute at low energy to the beta function. To take this fact into account, we divide the energy range from low energy scale, at which current experiments are conducted, to the grand unification scale, in two ranges. In the energy range from M_Z up to m only the SM particles contribute to the beta function. From m, at which the unparticle sector appears, to M_{GUT} we consider the effects of unparticle loops. The next step is to scan the parameters space of unparticles namely, the scale dimension d and the number of species n_s, to find a set of values for which the unification of gauge couplings is possible. We found that for a number of generations $n_s = 9$ and scale dimension $d = 1.5$, the unification of all three SM coupling is achieved at the energy scale $M_{GUT} = 10^{12}$. Results are depicted in Fig (5.7). It is notable that this unification scale value is orders of magnitude lower than the value $M_{GUT,Susy} = 10^{16}$ achieved using supersymmetric models.

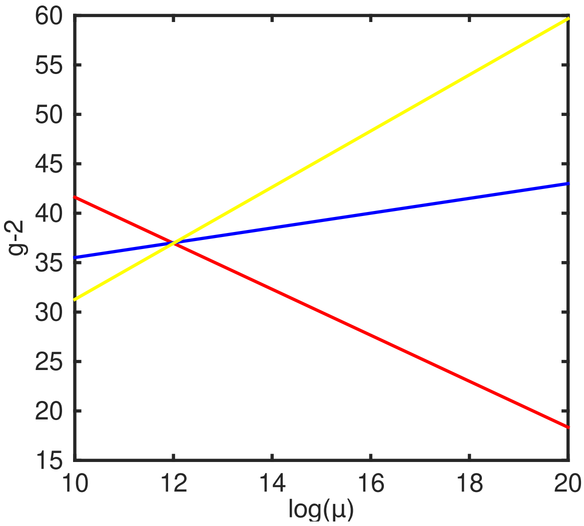

Figure 5.7: Unification of gauge coupling as a function of the energy scale μ. The red line represent hyper-charge coupling, blue line represent the weak coupling and the yellow line represent strong coupling. The three lines intersect at $M_{GUT} = 10^{12}$ for $n_s = 9$ and $d = 1.5$.

Chapter 6

Muon Anomalous Magnetic Moment in the Left-Right Symmetric Model

The work presented in this chapter represents an original contribution to the international conference of theoretical physics held in Constantine in October 2013. In this work we calculate the contribution of the left right symmetric model (LRSM) to the the muon anomalous magnetic moment (AMM). Specifically the effects of virtual loops of extra gauge bosons associated with the model. We found that for the chosen parameters values the AMM reduces from $2,6\sigma$ to 2.5σ which is small but not negligible. The deviation between the SM predictions for the muon AMM and experimental results, which persisted during the years, represents a strong hint for physics beyond the SM. Recently this discrepancy has received a further confirmation by the muon g-2 experiment at Fermi lab . The current value of the deviation sits at 4.2σ which is not high enough to declare a discovery, a 5σ is necessary for particle physics experiments. However, this new results prompted the search for new physics solutions.

Our work is based on the minimal left right symmetric model. This model has been proposed in the 1970s by Salam and pat [63] and later on by Mohabatra [64]. The original motivation for this model is to treat left handed fermions and their right handed counterparts on an equal footing. This is done by adding an additianal gauge group for right handed particles $SU(2)_R$. So the electroweak group becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Where here $B - L$ is the difference between baryons and lepton number. Mohabatra also proposed in [65] to restore parity symmetry at high energies, using spontaneous symmetry breaking as a mechanism, to explain the violation of left right symmetry at the electroweak scale. The symmetry breaking in this model takes place in two stages. In the first stage a right handed scalar triplet Δ_R break the left right symmetry down to SM symmetry

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \tag{6.1}$$

In the second stage the familiar symmetry breaking via, in this case, a bidoublet scalar field Φ looks like

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em} \tag{6.2}$$

A left triplet Δ_L is introduced to maintain parity symmetry into the theory. The three scalars are the following

$$\Delta_{L,R} = \begin{pmatrix} \Delta^+_{L,R}/\sqrt{2} & \Delta^{++}_{L,R} \\ \Delta^0_{L,R} & -\Delta^{++}_{L,R}/\sqrt{2} \end{pmatrix}, \; \Phi = \begin{pmatrix} \Phi^0_1 & \Phi^+_1 \\ \Phi^-_2 & \Phi^0_2 \end{pmatrix} \tag{6.3}$$

for each generator of the $SU(2)_R$ we associate a gauge boson which are W^1_R, W^2_R, W^3_R plus a vector field V^μ associated with U_{B-L} gauge group. The symmetry breaking induced by the vev of the Δ_R give masses to right gauge bosons but kepp left bosons W_L, Z_L massless. If we perform a rotation in the internal isospin space of the group $SU(2)_R \times U(1)_{B-L}$ we will get the neutral gauge boson Z_R as follows

$$\begin{pmatrix} Z^\mu_R \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & cos(\varphi) \end{pmatrix} \begin{pmatrix} W^\mu_R \\ V^\mu \end{pmatrix} \tag{6.4}$$

where φ is defined by

$$\cos(\varphi) = \frac{g_R}{\sqrt{g_R^2 + g_{B-L}^2}}, \; \sin(\varphi) = \frac{g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}} \tag{6.5}$$

where g_R is the coupling constant of the $SU(2)_R$ group and g_{B-L} is the coupling constant of the $U(1)_{B-L}$ group. The masses for right gauge bosons W_R and Z_R can be extracted from the kinetic term of the Higgs sector associated with Δ_R

$$\begin{aligned} \mathrm{Tr}\,|D^\mu \Delta_R|^2 &= \frac{g_R^2 V_R^2}{2} W^{\mu-}_R W^{\mu+}_R + \frac{V_R^2}{2}\left(g_R W^{\mu 3}_R - g_{B-L} V^\mu\right)\left(g_R W^{\mu 3}_R - g_{B-L} V^\mu\right) \\ &= \frac{g_R^2 V_R^2}{2} W^{\mu-}_R W^{\mu+}_R + \frac{V_R^2}{2} Z^\mu_R Z^\mu_R \end{aligned} \tag{6.6}$$

by the end of the first stage of SB, we end up with two charged right gauge bosons $W^{\mu\pm}_R$, one right neutral gauge boson Z_R, two neutral scalars H^0_i, two pseudo scalars φ^0_i, two singlet charged Higgs scalars H^+_i and two doubly charged scalars H^{++}_i, where $i = 1$ *or* 2. For more detailed study of the left right symmetric model see the review [66].

6.1 Calculation of the AMM in the LRSM

In our work, we use the LRSM to explain the anomalous magnetic moment of the muon (AMM). The study of AMM represents a very sensitive test of the SM at the quantum loop level and permits the investigation of physics that lie beyond it. The magnetic moment is defined as $\mu = g(\frac{e}{2m})s$, where g is the gyromagnetic ratio, s is the spin of the particle. The deviation of the magnetic moment from the value of the point-like Dirac particle ($g = 2$) is induced by the interactions of leptons with virtual particles which couple to electromagnetic field. Whereas, the electron anomaly provides the most precise measurement of the fine structure constant, the muon anomaly is more sensitive to virtual gauge bosons(because the muon mass is much larger than the electron mass). In this work, we consider all possible contributions from extra gauge bosons at the one loop level. Our purpose is to get a better interpretation of the experimental results of the muon anomaly.

6.1.1 Values of a_μ in the SM

The muon anomaly in the SM is the summation of three contributions

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{Had} \tag{6.7}$$

These contributions have been determined precisely in previews works. The QED contribution is the dominant one, and it has been calculated up to the fourth order α^4. The weak contribution has been calculated up to 3 loop level and it has not changed much in the last years. We present below the best results of the muon anomaly calculation in the SM [67]

$$a_\mu^{QED} = 11658471.958(0.143) \times 10^{-10} \tag{6.8}$$

$$a_\mu^{Weak} = 15.4(0.2) \times 10^{-10} \tag{6.9}$$

$$a_\mu^{Had} = 697.2(5.9) \times 10^{-10} \tag{6.10}$$

The total SM value for a_μ is

$$a_\mu^{QED} = 11659184.56(5.9) \times 10^{-10} \tag{6.11}$$

and the present experimental value for a_μ is

$$a_\mu^{Exp} = 11659208.56(6) \times 10^{-10} \tag{6.12}$$

thus, the deviation of the experimental value of the anomalous magnetic moment of the muon from the SM prediction is the following

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM} = 23.4(9) \times 10^{-10} \tag{6.13}$$

6.1.2 Calculation of a_μ

The LRSM contribution to the muon anomaly is done using one loop corrections from the heavy weak gauge bosons W_R and Z_R and the the neutral scalar and pseudo-scalar from the Higgs sector $H^0_{1,2}$ and $\varphi^0_{1,2}$. In our calculations we find that the most important contribution to the anomaly come from the weak gauge bosons W_R, Z_R. The contribution of the charged Higgs is negligible compared to other particles. The finals results are the following (for more details see appendix E)

$$a_\mu^{W_R} = \frac{\alpha}{16\pi\sin^2(\theta_W)}\left(\frac{m_\mu}{M_{W_R}}\right)^2 \times \frac{7}{3} + O\left(\left(\frac{m_\mu}{M_{W_R}}\right)\right) \tag{6.14}$$

$$a_\mu^{Z_R} = -\frac{\alpha m_\mu^2}{12\pi M_{Z_R}^2}\frac{1-\tan^2(\theta_W)(1+\tan^2(\theta_W))}{\sin^2(\theta_W)(1-\tan^2(\theta_W))} \tag{6.15}$$

$$a_\mu^{H_1^0} = \frac{\alpha}{8\pi\sin^2(\theta_W)}\frac{\tan^2(\beta)(1+\tan^2(\beta))}{1-\tan^2(\beta))}\left(\frac{m_\mu}{M_{W_L}}\right)^2\left(\frac{m_\mu}{M_{H_1^0}}\right)^2 \ln\left(\frac{M^2_{H_1^0}}{m_\mu^2}\right) \tag{6.16}$$

$$a_\mu^{H_2^0} = \frac{\alpha}{8\pi\sin^2(\theta_W)}\frac{(1+\tan^2(\beta))}{1-\tan^2(\beta))}\left(\frac{m_\mu}{M_{W_L}}\right)^2\left(\frac{m_\mu}{M_{H_2^0}}\right)^2 \ln\left(\frac{M^2_{H_2^0}}{m_\mu^2}\right) \tag{6.17}$$

$$a_\mu^{\varphi_1^0} = \frac{\alpha}{8\pi\sin^2(\theta_W)}\frac{\tan^2(\beta)(1+\tan^2(\beta))}{1-\tan^2(\beta))}\left(\frac{m_\mu}{M_{W_L}}\right)^2\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{M^2_{\varphi_1^0}}{m_\mu^2}} \tag{6.18}$$

$$a_\mu^{\varphi_2^0} = \frac{\alpha}{8\pi\sin^2(\theta_W)}\frac{(1+\tan^2(\beta))}{1-\tan^2(\beta))}\left(\frac{m_\mu}{M_{W_L}}\right)^2\int_0^1 dx\frac{x^3}{x^2+(1-x)\frac{M^2_{\varphi_2^0}}{m_\mu^2}} \tag{6.19}$$

where β is a parameter of the LRSM related to the expectation value for the bidoublet Φ

$$\tan(\beta) = \frac{\kappa_1}{\kappa_2}$$
$$\langle\Phi\rangle = \begin{pmatrix}\kappa_1 & 0\\ 0 & \kappa_2\end{pmatrix} \tag{6.20}$$

6.1.3 Numerical results

To calculate the effects of the LRSM spectrum on the AMM we use the following values for the LRSM parameters [68]: $M_{W_R} = M_{Z_R} = 1Tev$,$M_{H_1^0} =$

$M_{H_2^0} = M_{\varphi_1^0} = M_{\varphi_1^0} = 5Tev$, $\tan(\beta) = 10$. After summation of all contribution we get the result

$$a_\mu^{LR} = 0.137 \times 10^{-10} \tag{6.21}$$

the deviation of the experimental value of AMM of the muon from the SM prediction is reduced to

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM+LRSM} = 23.26(0.9) \times 10^{-10} \tag{6.22}$$

so we conclude that the LRSM with the current phenomenological constraints on its parameters, coming from direct production channels of the heavy weak bosons, and electroweak precision measurements, allows us to reduce slightly the deviation of the theoretical prediction of AMM from experiments from 2.6σ deviation in the SM to 2.5σ in the LRSM.

Chapter 7

Conclusion

Unparticle model, which started as a theoretical curiosity by Georgi and Co, ended up producing a reach theoretical and phenomenological consequences, ranging from dark matter and black holes to superconductivity. In this thesis we focused on construction of unparticle model charged under the electroweak standard model group $SU(2) \times U(1)$. We were able to derive vertex functions describing the interaction between the unparticle sector and the SM gauge bosons γ, W and Z, and derive their asymptotic forms in the large momentum limit, which dominates in loop integrals. Then we used these vertices to calculate the polarization function of γ, W and Z, with scalar and fermionic unparticle fields circulating in the loops. The results of polarization functions allowed us to compute the oblique parameters S and T in the scalar and fermionic cases. Then we used these results to find the region of parameter space, namely scale dimension d and the conformal breaking scale m, compatible with electroweak precision measurements. The results suggest that the CBS m must be inferior to 200 Gev, in the case of unfermion, for a renormalization scale value ranging from $\frac{M_Z}{2}$ to $2M_Z$. for scalar unparticles the allowed CBS values must be inferior to 350 Gev for the same renormalization scale interval, which enlarge the conformal window for unparticles assuming that the unparticle scale is $\Lambda_{\mathcal{U}} \sim 1Tev$. These results implies that unparticles effects should be detectable in the energy range $m \leq 200\ Gev \leq E \leq \Lambda_{\mathcal{U}}$ for unfermion and $m \leq 350\ Gev \leq E \leq \Lambda_{\mathcal{U}}$ for scalar unparticles.

In the course of this thesis we also used polarization functions to estimate the effects on the running of gauge coupling induce by virtual unparticle fields with spin 0. Contrary to other works in the literature, in this case we considered unparticle fields that are charged under all three SM gauge groups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$. We found that the unification between

the three gauge coupling g_S,g and g' requires a large number of unparticle species $n_s = 9$ for the scale dimension value $d = 1.5$. The unification scale attained through unparticle contribution is $M_{GUT} = 10^{12}$ which is orders of magnitude lower then the value reached using supersymmetric models.
In the last part of this thesis we presented a computation of the muon anomalous magnetic moment of the muon, at the one loop level, based on the left right symmetric model. We found that the contribution of the extras gauge bosons and the Higgs sector decreases the discrepancy between the SM prediction and experimental results from 2.6σ deviation to 2.5σ which is small but non negligible contribution

In future works we hope to use the Ads/CFT correspondence, mentioned briefly in chapter 4, to achieve a deeper understanding of unparticle physics. For example unparticle hadronization process which is related to strong coupling conformal field theory. In addition to that we aim to construct a gauged model for vector and tensor unparticle which is absent in the literature.

Appendix A

Loop Integrals

There is multiple ways to perform loop integrals, in our work, we use Feynman parametrization and dimensional regularization. The general form of Feynman parametrization is the following

$$\prod_{i=1}^{n} \frac{1}{A_i^{\alpha_i}} = \frac{\Gamma\left(\sum_{i=1}^{n} \alpha_i\right)}{\prod_{i=1}^{n} \Gamma(\alpha_i)} \int_0^1 \frac{\prod_{i=1}^{n} dx_i x_i^{\alpha_i - 1}}{\left(\sum_{i=1}^{n} A_i x_i\right)^{\sum_{i=1}^{n}}} \delta\left(1 - \sum_{i=1}^{n} x_i\right) \tag{A.1}$$

where A_i are arbitrary complex numbers, in our case the denominator of the propagators inside the loops. Γ is the gamma function defined by the integral

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt \tag{A.2}$$

$\Gamma(z)$ satisfies the following proprieties

$$\begin{aligned} \Gamma(z+1) &= z\Gamma(z) \\ \Gamma(n+1) &= n! \end{aligned} \tag{A.3}$$

the gamma function admits poles for negative integer values $z = 0, -1, -2, \ldots$. In the vicinity of the pole $z = m$ we have the following approximation

$$\Gamma(z) = \frac{(-1)^m}{m!} \frac{1}{m+z} + \frac{(-1)^m}{m!} \psi(m+1) + O(m+z) \tag{A.4}$$

where ψ is the logarithmic derivative of Γ

$$\psi(z) = \frac{d}{dz} \ln(\Gamma(z)) \tag{A.5}$$

which have the following proprieties

$$\begin{aligned} \psi(1) &= -\gamma \\ \psi(z+1) &= \psi(z) + \frac{1}{z} \end{aligned} \tag{A.6}$$

where $\gamma = 0.5772156649\ldots$ is the Euler constant. From eq(A.4), we conclude that for $\epsilon \to 0$

$$\Gamma(\epsilon) = \frac{1}{\epsilon} + \psi(1) + O(\epsilon)$$
$$\Gamma(-n+\epsilon) = \frac{(-1)^D}{n!}\left(\frac{2}{\epsilon} + \psi(n+1) + 1\right) \tag{A.7}$$

with the help of these proprieties of the gamma function we can perform loop integrals and separate the divergent and finite parts. Other useful formulas are the Euler function, $B(\alpha, \beta)$ defined as

$$B(\alpha,\beta) = \int_0^1 dx x^\alpha (1-x)^\beta = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} \tag{A.8}$$

and the generalized hypergeometric functions ${}_2F_1$,${}_3F_2$,${}_4F_3$ defined as

$$ {}_{q+1}F_q(a_1,\ldots,a_{q+1};b_1,\ldots,b_q;z) = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{q+1}(a_i)_n}{\prod_{i=1}^{q}(b_i)_n}\frac{z^D}{n!}$$
$$= \left(\prod_{i=1}^{q}\frac{\Gamma(b_i)}{\Gamma(a_i)\Gamma(b_i-a_i)}\right)\int_0^1\cdots\int_0^1\prod_{i=1}^{q} t_i^{a_i-1}(1-t_i)^{b_i-a_i-1}(1-z\prod_{i=1}^{q} t_i)^{-a_i+1}dt_1\ldots dt_q \tag{A.9}$$

$$Re(b_i) > Re(a_i) > 0 \wedge 1 \leq i \leq q \wedge |arg(1-z)| \leq \pi \tag{A.10}$$

where the second equation is the integral representation of the generalized hypergeometric function${}_{q+1}F_q$. $(a_i)_n$ is the Pochhammer symbol defined as

$$(a_i)_n = \frac{\Gamma(a_i+n)}{\Gamma(n)} \tag{A.11}$$

A.1 Evaluation of the integral $I(k,\alpha,\beta)$ from section (5.2)

Next we evaluate the integrals $I(k,\alpha,\beta)$ from eq(5.81) section(5.2). All the loop integrals encountered in the course of this thesis can be reduced to one of the following form

$$I(k,\alpha,\beta) = \int \frac{d^D p}{(2\pi)^D}\frac{(p^2)^k}{(p^2-m^2)^\alpha((p+q)^2-m^2)^\beta} \tag{A.12}$$

We will use dimensional regularization to compute loop integrals. In this method integrals of the type (A.12) can be calculated by using Feynman parametrization method relaying on these basic formulas

$$\begin{aligned}
\int \frac{d^Dp}{(2\pi)^D}\frac{1}{(p^2-\Delta^2)^\alpha} &= \frac{i(-1)^\alpha}{(4\pi)^{n/2}}\frac{\Gamma(\alpha-n/2)}{\Gamma(\alpha)}(\Delta^2)^{n/2-\alpha} \\
\int \frac{d^Dp}{(2\pi)^D}\frac{p^2}{(p^2-\Delta^2)^\alpha} &= \frac{i(-1)^{\alpha-1}}{(4\pi)^{n/2}}\frac{D}{2}\frac{\Gamma(\alpha-n/2-1)}{\Gamma(\alpha)}(\Delta^2)^{n/2-\alpha+1} \\
\int \frac{d^Dp}{(2\pi)^D}\frac{p^\mu p^\nu}{(p^2-\Delta^2)^\alpha} &= \frac{i(-1)^{\alpha-1}}{(4\pi)^{n/2}}\frac{\eta^{\mu\nu}}{2}\frac{\Gamma(\alpha-n/2-1)}{\Gamma(\alpha)}(\Delta^2)^{n/2-\alpha+1}
\end{aligned} \quad (A.13)$$

the result for the integral (A.12) is the following

$$\begin{aligned}
I(k,\alpha,\beta) &= \frac{1}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(\frac{D}{2}+k)\Gamma(\alpha+\beta-\frac{D}{2}-k)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\frac{D}{2})}(m^2)^{\frac{D}{2}+k-\alpha-\beta} \\
&\times \int_0^1 dx x^{\alpha-1}(1-x)^{\beta-1}\left(1-\frac{q^2}{m^2}x(1-x)\right)^{\frac{D}{2}+k-\alpha-\beta}
\end{aligned} \quad (A.14)$$

we will assume $\tau = \dfrac{q^2}{m^2} < 1$ which is consistent with experimental data since q^2 will be in the vicinity of M_Z^2 and we will assume that m the CBS is superior to M_Z. To calculate the integral (A.14) we affect a Taylor expansion for the coefficient $(1-\dfrac{q^2}{m^2}x(1-x)^{\frac{D}{2}+k-\alpha-\beta}$ with respect to $\tau = \dfrac{q^2}{m^2}$. The result is the following

$$(1-\tau x(1-x))^{\frac{D}{2}+k-\alpha-\beta} = \sum_{j=0}^{\infty}\frac{1}{j!}\frac{\Gamma\left(1-\left(\frac{D}{2}+k-\alpha-\beta\right)+j\right)}{\Gamma\left(1-\left(\frac{D}{2}+k-\alpha-\beta\right)\right)}x^j(1-x)^j\tau^j \quad (A.15)$$

substituting eq(A.15) in (A.14) we find

$$\begin{aligned}
I(k,\alpha,\beta) &= \frac{1}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(\frac{D}{2}+k)\Gamma(\alpha+\beta-\frac{D}{2}-k)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\frac{D}{2})}(m^2)^{\frac{D}{2}+k-\alpha-\beta} \\
&\times \int_0^1 dx x^{\alpha-1}(1-x)^{\beta-1}\sum_{j=0}^{\infty}\frac{1}{j!}\frac{\Gamma\left(1-\left(\frac{D}{2}+k-\alpha-\beta\right)+j\right)}{\Gamma\left(1-\left(\frac{D}{2}+k-\alpha-\beta\right)\right)}x^j(1-x)^j\tau^j
\end{aligned} \quad (A.16)$$

using the definition of the beta function eq(A.8) and the following propriety of the gamma functions

$$\Gamma(2\alpha) = 2^{2\alpha-1}\pi^{-\frac{1}{2}}\Gamma(\alpha)\Gamma(\alpha+\frac{1}{2}) \quad (A.17)$$

we arrive at the result

$$\begin{aligned}I(k,\alpha,\beta) &= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\tau}{4}\right)^j\frac{(1+\alpha+\beta-k,j)\,(\alpha,j)(\beta,j)}{\left(\frac{\alpha+\beta}{2},j\right)\left(\frac{\alpha+\beta+1}{2},j\right)}\\ &= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}{}_3F_2\left(\left\{1,\alpha+\beta-\frac{D}{2}-k,\alpha,\beta\right\},\left\{\frac{\alpha+\beta}{2},\frac{\alpha+\beta+1}{2}\right\},\frac{\tau}{4}\right)\end{aligned} \tag{A.18}$$

${}_3F_2$ is a Hypergeometric function defined in eq(A.9) which converge for $\tau = \frac{q^2}{m^2} < 1$. In using Feynman parametrization we made the substitution $p = p-q(1-x)$ in the denominator of the integrals (5.68,5.76) a similar substitution must take place for the numerators Eq(5.69-5.74) and Eq(5.77-5.79) as well. Taking these changes into account we can express the polarization functions (5.56) and (5.57) in terms of hypergeometric functions like the one in (A.9) for each integral in (5.68) and (5.76). So the polarization function expressed by the sum of diagram (a) and (b) in Fig (5.1) ($\Pi^{ab} = \Pi^{ab}_{(a)} + \Pi^{ab}_{(b)}$) is given by

$$\begin{aligned}\Pi^{ab} =& -8i\frac{g_a g_b}{(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}-2}\Bigg\{\Gamma(2-\frac{D}{2})\Big\{F_1+(\frac{3}{2}-d)BF_2+(\frac{3}{2}-d)^2BF_4\Big\}\\ &+\Gamma(3-\frac{D}{2})(\frac{3}{2}-d)^2BF_5\Bigg\} - 4i\frac{g^2}{(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}-2}\frac{\Gamma(2-\frac{D}{2})}{1-\frac{D}{2}}\,\mathrm{Tr}(T_1)F_6\end{aligned} \tag{A.19}$$

The result (A.19) is not renormalizable. To find the renormalized polarization function $\Pi^{ab}_{ren}(q^2)$, we use the $\overline{MS}$ renormalization scheme. In this scheme we replace the spacetime dimension D by $4-2\epsilon$ in all the terms of (A.19) that contain D. Then, we extract the terms proportional to $\Gamma(\epsilon)$. these terms diverge when we take the limit $\epsilon \to 0$. After that, we make a Taylor expansion to first order for terms like $a^{\epsilon} \approx 1-\ln(a)\epsilon$. Then, we isolate the ones proportional to $\Gamma(\epsilon)$. and we make the substitution

$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon^2) \tag{A.20}$$

For terms proportional to hypergeometric functions ${}_2F_1$ and ${}_2F_3$ we have two possibilities, either we have terms of the form $\alpha\ {}_3F_2$ where α is independent from $\Gamma(\epsilon)$, or we have terms of the form $\Gamma(\epsilon)_3F_2$. For the first type we find

$$\begin{aligned}\alpha\ {}_3F_2(\{\epsilon,a,b\}\{e,f\};\frac{\tau}{4}) &= \alpha\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\tau}{4}\right)^j\frac{(\epsilon,j)\,(a,j)(b,j)}{(e,j)\,(f,j)}\\ &= \alpha\frac{\Gamma(e)\Gamma(f)}{\Gamma(a)\Gamma(b)\Gamma(\epsilon)}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\tau}{4}\right)^j\frac{\Gamma(\epsilon+j)\Gamma(a+j)\Gamma(b+j)}{\Gamma(e+j)\Gamma(f+j)}\\ &= \alpha+\frac{\Gamma(e)\Gamma(f)}{\Gamma(a)\Gamma(b)\Gamma(\epsilon)}\sum_{j=1}^{\infty}\frac{1}{j!}\left(\frac{\tau}{4}\right)^j\frac{\Gamma(\epsilon+j)\Gamma(a+j)\Gamma(b+j)}{\Gamma(e+j)\Gamma(f+j)} \qquad \text{(A.21)}\end{aligned}$$

If we take the limit $\epsilon \to 0$ we get $\Gamma(\epsilon) \to \infty$, so the sum in eq(A.21) vanishes and we end up with

$$\lim_{\epsilon\to 0}\alpha_3F_2(\{\epsilon,a,b\}\{e,f\};\frac{\tau}{4}) = \alpha \qquad \text{(A.22)}$$

for terms of the form $\Gamma(\epsilon)_3F_2$ we have

$$\Gamma(\epsilon)_3F_2(\{\epsilon,a,b\}\{e,f\};\frac{\tau}{4}) = \Gamma(\epsilon)+\frac{\Gamma(e)\Gamma(f)}{\Gamma(a)\Gamma(b)}\sum_{j=1}^{\infty}\frac{1}{j!}\left(\frac{\tau}{4}\right)^j\frac{\Gamma(\epsilon+j)\Gamma(a+j)\Gamma(b+j)}{\Gamma(e+j)\Gamma(f+j)} \qquad \text{(A.23)}$$

taking the limit $\epsilon \to 0$ for the second term in eq(A.23) and making the substituting $j = j'+1$ we get the following result

$$\begin{aligned}\Gamma(\epsilon)_3F_2(\{\epsilon,a,b\}\{e,f\};\frac{\tau}{4}) &= \Gamma(\epsilon)+\frac{\Gamma(e)\Gamma(f)}{\Gamma(a)\Gamma(b)}\\ &\times\sum_{j'=0}^{\infty}\frac{1}{(j'+1)!}\left(\frac{\tau}{4}\right)^{j'+1}\frac{\Gamma(1+j)\Gamma(a+1+j')\Gamma(b+1+j')}{\Gamma(e+1+j')\Gamma(f+1+j')}\\ &= \Gamma(\epsilon)+\frac{\Gamma(e)\Gamma(f)}{\Gamma(a)\Gamma(b)}\frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(e+1)\Gamma(b+1)}\frac{\tau}{4}\sum_{j'=0}^{\infty}\frac{1}{(j')!}\left(\frac{\tau}{4}\right)^{j'}\frac{(1,j')(1,j')(a+1,j')(b+1,j')}{(2,j')(e+1,j')(f+1,j')}\\ &= \Gamma(\epsilon)+\frac{ab}{ef}\ {}_4F_3\left(\{1,1,a+1,b+1\},\{2,e+1,f+1\},\frac{\tau}{4}\right) \qquad \text{(A.24)}\end{aligned}$$

using both eq (A.22) and (A.24) we can renormalize the polarization function eq(A.19). We will find a sum of terms proportional to $\Gamma(\epsilon)$ and terms independent from $\Gamma(\epsilon)$. We will get rid off the former and keep the rest. The renormalized expression is given in the main text eq(5.88)

Appendix B

Calculation of the constants A,B,D from chapter (5)

In this appendix we calculate the constants that appear in the calculation of the polarization function $\Pi^{\gamma\gamma}$,Π^{ZZ} and $\Pi^{Z\gamma}$. We begin with $\Pi^{\gamma\gamma}$, in this case the coupling constants are

$$g_a = g_b = \frac{e}{2}$$

where e is the electromagnetic coupling constant. We have from eqs(5.70-5.75)

$$A = \mathrm{Tr}\big(T_1^a T_1^b - T_2^a T_2^b\big)$$
$$B = \mathrm{Tr}\big(T_1^a T_1^b + T_2^a T_2^b\big)$$
$$D = \mathrm{Tr}\big(3T_1^a T_1^b + T_2^a T_2^b\big)$$

since electromagnetic interaction conserves parity, the coefficient T_2 proportional to γ_5 is zero. So, we get

$$A = B = \mathrm{Tr}\big(T_1^a T_1^b\big)$$
$$D = 3\,\mathrm{Tr}\big(T_1^a T_1^b\big)$$

for electromagnetic interactions the generator $T_1^a = T_1^b = Q$, where Q is the charge operator given by

$$Q = \begin{pmatrix} q_u & 0 \\ 0 & q_d \end{pmatrix}$$

q_u, q_d are the charges of the upper and lower components, respectively of the unparticle multiplet introduced in eq(5.35). So we conclude

$$A = B = \mathrm{Tr}\big(Q^2\big) = q_u^2 + q_d^2$$
$$D = 3(q_u^2 + q_d^2)$$

In addition to A,B,D we should calculate $\mathrm{Tr}(T_1)$ related to the loop integral involving two gauge bosons and two unparticles, we find

$$\mathrm{Tr}(T_1) = Q^2 = q_u^2 + q_d^2$$

B.1 Constants for Π^{ZZ}

In this case the coupling constants are

$$g_a = g_b = \frac{e}{4\sin(\theta_W)\cos(\theta_W)}$$

the generators of the $Z\mathcal{U}$ coupling are the following

$$T_1^a = T_1^b = \frac{\sigma_3}{2} - 2\sin^2(\theta_W)Q$$
$$T_2^a = T_2^b = -\frac{\sigma_3}{2}$$

so A, B and D are given by

$$A = \mathrm{Tr}\big(T_1^a T_1^b - T_2^a T_2^b\big) = \mathrm{Tr}\left(2\sin^2(\theta_W)\begin{pmatrix} 2\sin^2(\theta_W)q_u^2 - q_u & 0 \\ 0 & 2\sin^2(\theta_W)q_d^2 + q_d\end{pmatrix}\right)$$
$$= 2\sin^2(\theta_W)(q_d - q_u + 2\sin^2(\theta_W)(q_u^2 + q_d^2))$$

$$B = \mathrm{Tr}\big(T_1^a T_1^b + T_2^a T_2^b\big) = \mathrm{Tr}\left(\frac{1}{4}\begin{pmatrix} (1-4\sin^2(\theta_W)q_u)^2 + 1 & 0 \\ 0 & (1+4\sin^2(\theta_W)q_d)^2 + 1\end{pmatrix}\right)$$
$$= 1 + 2\sin^2(\theta_W)(q_d - q_u) + 4\sin^4(\theta_W)(q_u^2 + q_d^2)$$

$$D = \mathrm{Tr}\big(3T_1^a T_1^b + T_2^a T_2^b\big) = \mathrm{Tr}\left(\frac{3}{4}\begin{pmatrix} (1-4\sin^2(\theta_W)q_u)^2 & 0 \\ 0 & (1+4\sin^2(\theta_W)q_d)^2\end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}\right)$$
$$= 2 + 6\sin^2(\theta_W)(q_d - q_u) + 12\sin^4(\theta_W)(q_u^2 + q_d^2)$$

and

$$T_1 = \frac{1}{4}\left(\sigma_3^2 + 8\sin^4(\theta_W)Q^2 - 4\sigma_3 Q\sin^2(\theta_W)\right)$$

which gives

$$\mathrm{Tr}(T_1) = \frac{1}{2}\left(1 + 4\sin^4(\theta_W)(q_u^2 + q_d^2) + 2\sin^2(\theta_W)(q_d - q_u)\right)$$

B.2 Constants for $\Pi^{Z\gamma}$

The coupling constants in this case are the following

$$g_a = \frac{e}{2},\ g_b = \frac{e}{4\sin(\theta_W)\cos(\theta_W)}$$

and the generators for one unparticle one gauge boson interactions are given by

$$T_1^a = Q,\ T_2^a = 0$$

for $\gamma\mathcal{U}$ vertex, and

$$T_1^b = \frac{\sigma_3}{2} - 2\sin^2(\theta_W)Q,\ T_2^b = -\frac{\sigma_3}{2}$$

for $\gamma\mathcal{U}$ vertex. So, we get

$$\begin{aligned} A &= \mathrm{Tr}\big(T_1^aT_1^b - T_2^aT_2^b\big) = \mathrm{Tr}\big(T_1^aT_1^b\big) \\ &= \frac{1}{2}\left(q_u - q_d - 4\sin^2(\theta_W)(q_u^2 + q_d^2)\right) \end{aligned}$$

and

$$B = A = \frac{1}{2}\left(q_u - q_d - 4\sin^2(\theta_W)(q_u^2 + q_d^2)\right)$$

and

$$D = 3A = \frac{3}{2}\left(q_u - q_d - 4\sin^2(\theta_W)(q_u^2 + q_d^2)\right)$$

the generator corresponding to $Z\gamma\mathcal{U}\mathcal{U}$ interaction is

$$T_1 = Q\frac{\sigma_3}{2} = \frac{1}{2}\begin{pmatrix} q_u & 0 \\ 0 & -q_d \end{pmatrix}$$

So, we get

$$\mathrm{Tr}(T_1) = \frac{1}{2}(q_u - q_d)$$

Appendix C

article

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Constraints on electroweak gauged unparticle model from the oblique parameters S and T

H. Taibi* and N. Mebarki†

Laboratoire de Physique Mathématique et Subatomique, Université Mentouri,
Constantine 1, Rue de Ein Elbey, Constantine 25000, Algeria
**taibi.hamza@hotmail.com*
†nnmebarki@yahoo.fr

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We calculate the one loop contribution from an unparticle gauged model, based on the group $\mathrm{SU}(2) \times \mathrm{U}(1)$, to the electroweak oblique parameters S and T. Using the current bounds on S and T from electroweak measurements, we find the constraints on the scale dimension d of the unparticle fermionic fields to be $1.5 < d < 1.7$ and the constraints on the conformal breaking scale m to be $100 < m < 200$ Gev. The bounds on m impose a lower limit on the conformal window of the unparticle fields which means that unparticle are not detectable below 100 Gev.

Keywords: Unparticles; gauged model; oblique parameters.

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1. Introduction

The Standard Model (SM) has been so far in excellent agreement with experiment. However, it fails to explain neutrino oscillations, dark matter, and the origin of baryon asymmetry in the universe. Moreover, the hierarchy problem indicates that the SM in its basic version cannot describe physics above the weak scale. These inconsistencies and shortcomings of the SM prompted the study of physics beyond the Standard Model (BSM). A particularly interesting model of BSM proposed about a decade ago is unparticle model,[1] which describes a scale invariant hidden sector interacting with SM particles at high energy via messenger particles. These interactions are organized in an effective field theory in which unparticles are represented by scale invariant operators. An extension of the unparticle model to include

*Corresponding author.

operators with quantum numbers was introduced in Ref. 2. For any new physics model to be valid, it must be consistent with the SM predictions. In this regard, the electroweak precision tests represent a powerful tool to check the compatibility of a new model with experimental data. To achieve this goal for the unparticle model, we consider unparticle fields embodied in the SM electroweak group. These fields would induce loop effects on the electroweak precision tests, represented as contributions to the oblique parameters S and T.[3]

In Sec. 2, we give a short review of gauged unparticle model and we calculate its contributions to the oblique parameters S and T. In Sec. 3, we use the results of the previous section to study the parameters space of unparticles, and finally a short summary and conclusion are given.

2. The Model

The purpose of our paper is to calculate the effects of unparticles sector on electroweak observables. For this reason, we must find Feynman vertices describing the interactions of unparticle fields with the electroweak SM gauge bosons.

The unparticle stuff are described by scale invariant fields with scaling dimension d. Conformal invariance impose a particular form for the green function of unparticles. The free propagator of fermionic unparticles in momentum space is:

$$\Delta_{U_f}(p,\mu) = \frac{A(d-1/2)}{2cos(\pi d)} \frac{i}{(\not{p}-m)\Sigma_0(p)} \tag{1}$$

where $\Sigma_0(p) = (m^2 - p^2)^{3/2-d}$, p is the momentum, m is the conformal symmetry breaking scale, and A is a normalization factor defined by:

$$A(d) = \frac{16\pi^{3/2}}{(2\pi)^2 d)} \frac{\Gamma(d+1/2)}{\Gamma(d-1)\Gamma(2d)} \tag{2}$$

with $3/2 \leq d \leq 5/2$. In order to incorporate the unparticle fields to the SM gauge group, we use the following action:

$$S = \int d^4x\, d^4y \Big(\mathcal{U}_L^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) \mathcal{W}_L(x,y) \mathcal{U}_L + \mathcal{U}_R^\dagger(x) \tilde{\Delta}_{\mathcal{U}}^{-1}(x-y) \mathcal{W}_R(x,y) \mathcal{U}_R \Big) \tag{3}$$

where $\mathcal{U}_L$ is the left-handed unparticle multiplet, which transform according to the gauge group $\mathrm{SU}(2)_L$. $\mathcal{U}_R$ is the right-handed $\mathrm{SU}(2)_L$ singlet, which transform according to the hypercharge group $\mathrm{U}(1)_Y$. To ensure gauge invariance, we have introduced the Wilson line $\mathcal{W}(x,y)$ defined as:

$$\mathcal{W}_L(x,y) = P \exp\left(\int_x^y (T^a W_\mu^a - ig' Y B_\mu) du^\mu \right), \tag{4}$$

$$\mathcal{W}_R(x,y) = \exp\left(\int_x^y -ig' Q B_\mu\, du^\mu \right), \tag{5}$$

where P denotes path ordering in the generators T^a in the unparticle representation. Q is the charge operator in the same representation. To find the interaction vertices of unparticles with physical gauge bosons Z, W and γ of the SM, we replace W^a, B in Eqs. (4) and (5) according to the relations:

$$W_3^\mu = \cos(\theta_W)Z^\mu + \sin(\theta_W)A^\mu, \tag{6}$$

$$B^\mu = -\sin(\theta_W)Z^\mu + \sin(\theta_W)A^\mu, \tag{7}$$

$$W^\mu = (W_1 + iW_2)/\sqrt{2}, \quad W^{\mu\dagger} = (W_1 - iW_2)/\sqrt{2} \tag{8}$$

where θ_W is the Weinberg mixing angle.

Now, using the same techniques developed by Terning *et al.*, in the context of nonlocal chiral quark model (see Ref. 4), we derive Feynman vertices for the coupling of unparticles with one and two gauge bosons as follows

$$\begin{aligned} ig\Gamma^\mu(p,q) &= i\frac{\delta^3 S}{\delta V^\mu(q)\delta\mathcal{U}_L^\dagger(p+q)\delta\mathcal{U}(p)} \\ &= i\frac{g}{2}(\gamma^\mu(T_a + T_b\gamma_5)(\Sigma_0(p) + \Sigma_0(p+q)) \\ &\quad + (2\not{p} + \not{q} - 2m)(T_a + T_b\gamma_5)(2p+q)^\mu\Sigma_1(p,q)). \end{aligned} \tag{9}$$

As a check, we note that this vertex satisfies the Ward–Takahashi identity[5]

$$iq_\mu\Gamma^\mu = \Delta^{-1}(p+q,m)T^a - T^a\Delta^{-1}(p,m). \tag{10}$$

We will also need the explicit form of the vertex involving two unfermions and two gauge bosons:

$$\begin{aligned} & ig^a g^b\Gamma^{ab\mu\nu}(p,q_1,q_2) \\ &\quad = i\frac{\delta^3 S}{\delta V_\mu^a(q_1)V_b^\nu(q_2)\delta\mathcal{U}_L^\dagger(p+q_1+q_2)\delta\mathcal{U}(p)} \\ &\quad = i\frac{g_a g_b}{2}\Big((2\not{p} + \not{q_1} + \not{q_2} - 2m)\big[\{T^a,T^b\}g^{\mu\nu}\Sigma_1(p,q_1+q_2) \\ &\quad + T^aT^b(2p^\mu + 2q_2^\mu + q_1^\mu)(2p^\nu + q_2^\nu)\Sigma_2(p,q_2,q_1) + T^bT^a(2p^\mu + q_1^\mu) \\ &\quad \times (2p^\mu + 2q_1^\mu + q_2^\mu)\Sigma_2(p,q_1,q_2)\big] + \gamma^\mu\Gamma^{ab\nu}(p,q_2,q_1) + \gamma^\nu\Gamma^{ab\mu}(p,q_1,q_2)\Big). \end{aligned} \tag{11}$$

g and $g_{a,b}$ denote unparticle coupling with SM gauge bosons, T_a and T_b are operators defined in the unparticle representation and the form factors are

$$\Sigma_1(p,q) = \frac{\Sigma_0(p+q) - \Sigma_0(p)}{(p+q)^2 - p^2}, \tag{12}$$

$$\Sigma_2(p,q_1,q_2) = \frac{\Sigma_1(p,q_1+q_2) - \Sigma_1(p,q_1)}{(p+q_1+q_2)^2 - (p+q_1^2)} \tag{13}$$

and $\Gamma^{ab\mu}$ is defined as

$$\Gamma^{ab\mu} = T^bT^a(2p^\mu + q_1^\mu)\Sigma_1(p,q_2) + T^bT^a(2p^\mu + 2q_2 + q_1^\mu)\Sigma_1(p+q_2,q_1). \tag{14}$$

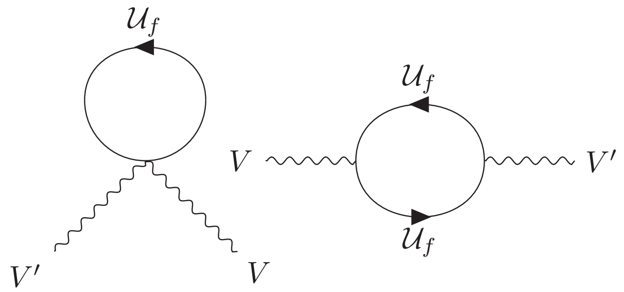

Fig. 1. The one loop contribution to polarization functions from charged fermionic unparticle fields, V and V' stand for γ, Z or W.

For the Abelian group U(1), it is sufficient to replace T_a with 1 and T_b with 0. For W and Z we define T_a and T_b as follows

$$\text{for } W: \; T_a = \frac{\sigma^-}{2}, T_b = \frac{\sigma^+}{2}, \tag{15}$$

$$\text{for } Z: \; T_a = \frac{\sigma^3}{2} - 2\sin^2(\theta)Q, T_b = -\frac{\sigma^3}{2}, \tag{16}$$

σ^-, σ^+ and σ^3 are Pauli matrices.

Now that we have derived Feynman vertices we can calculate the unparticles contribution to the oblique parameters S and T. The explicit expressions of these parameters are the following

$$S = \frac{4s_w^2 c_w^2}{\alpha}\left(\frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} - \frac{c_w^2 - s_w^2}{s_w c_w}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right) \tag{17}$$

and

$$T = \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right] \tag{18}$$

$\Pi_{ab}(q^2)$, where a, b stand for γ, Z or W, denote the new physics contribution to the part proportional to the metric $g_{\mu\nu}$ of the self-energies functions $\Pi^{\mu\nu}_{ab}(q^2) = ig_{\mu\nu}\Pi_{ab}(q^2) + \cdots$. The derivatives $\Pi'_{ab}(q^2)$ are defined by $\Pi'_{ab}(q^2) = d\Pi_{ab}(q^2)/dq^2$. α is the fine structure constant and $s_w = \sin(\theta_W)$, $c_w = \cos(\theta_W)$.

In Fig. 1, we show a typical diagram of the fermionic unparticles loops contributions to the self-energy functions $\Pi_{ab}(q^2)$ at the one loop level, where V and V' stand for γ, Z or W. The complicated Feynman vertices (9 and 11) and the nonstandard form of the propagator (1) do not allow the application of the usual tensor reduction recipe to compute the integrals corresponding to the diagrams in Fig. 1. However, if we look at the large p region of the loop integral, as is done in Refs. 6 and 7, we can affect a Taylor expansion of the function $\Sigma(p+q)$ for small q.

$$\Sigma(p' - q) = \Sigma(p')\left(1 - \frac{3/2 - d}{p'^2 - m^2}(q^2 - 2q\cdot p') + \frac{(3/2-d)(1/2-d)}{(p'^2 - m^2)^2}(q^2 - 2q\cdot p')^2 + \cdots\right) \tag{19}$$

where $p' = p + q$. Taking the first order in the expansion coefficient $y = q - 2q \cdot p'$ the form factors Σ_1 and Σ_2, defined in Eqs. (12 and 13), become

$$\Sigma_1(p,q) \simeq \frac{(-1)^{3/2-d}(3/2-d)}{((p+q)^2 - m^2))^{d-1/2}} + \cdots \tag{20}$$

and

$$\Sigma_2(p,q_1,q_2) \simeq \frac{(-1)^{5/2-d}(3/2-d)(d-1/2)}{((p+q_1)^2 - m^2)} \times \frac{1}{((p+q_1+q_2)^2 - m^2)^{d-1/2}} + \cdots . \tag{21}$$

Using Eqs. (20) and (21) in the calculations of the loop integrals contained in the polarization functions of Eqs. (17) and (18), we find the one loop contribution of unfermions to the oblique parameters S and T as follows

$$T = \frac{m^2}{8\pi s_w^2 c_w^2 M_Z^2}\Bigg\{\frac{23}{6} + 4d - 2d^2 + (1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)) \times \left(-\frac{7}{2} + 6d - \frac{10}{3}d^2 - \frac{2}{3}\left(\frac{3}{2} - d\right)^2 \frac{1 + 3s_w^2(q_d - q_u) + 6s_w^4(q_u^2 + q_d^2)}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)}\right) + \ln\left(\frac{\mu^2}{m^2}\right)\left(-\frac{5}{4} + \frac{14}{2}d - 4d^2 + (1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)) \times \left(43 - 44d + 12d^2 + \frac{4s_w^2((q_d - q_u) + 2s_w^2(q_u^2 + q_d^2))}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2}\right)\right)\Bigg\} \tag{22}$$

and

$$S = -\frac{(1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2))}{2\pi}\left(F_1 + F_2 + \ln\left(\frac{\mu^2}{m^2}\right)(F_3 + F_4 + F_5)\right) + \frac{c_w^2 - s_w^2}{48\pi}\left(\frac{q_d - q_u + 2s_w^2(q_u^2 + q_d^2)}{70}(6115 - 6219d + 568d^2 + 420d^3) + 8(q_d - q_u)(3/2 - d)^2(d - 1/2) + \ln\left(\frac{\mu^2}{m^2}\right)(q_d - q_u - 4s_w^2(q_u^2 + q_d^2)) \times (-16 - 33d + 28d^2 - 4d^3)\right) + \frac{s_w^2 c_w^2(q_u^2 + q_d^2)}{24\pi} \times \left(\frac{7123 - 4959d + 848d^2 - 140d^3}{70} + \ln\left(\frac{\mu^2}{m^2}\right)(-16 - 33d + 28d^2 - 4d^3)\right) \tag{23}$$

with

$$F_1 = \frac{1}{3}\left(d-\frac{1}{2}\right)^2\left(\frac{5}{2}-d\right){}_4F_3\left(1,1,d+\frac{1}{2},\frac{7}{2},2,2,\frac{5}{2},\frac{\tau}{4}\right)+\frac{1}{4}\left(d+\frac{1}{2}\right)\left(\frac{5}{2}-d\right)$$
$$\times\left(\frac{3}{2}-d\right){}_4F_3\left(1,1,d+\frac{3}{2},\frac{7}{2}-d,2,3,\frac{5}{2},\frac{\tau}{4}\right)+\left(\frac{3}{2}-d\right)^2$$
$$\times\left(\left(\frac{4}{3}\frac{m^2}{M_Z^2}-\frac{11}{24}\right){}_2F_1\left(1,2,\frac{5}{2},\frac{\tau}{4}\right)-\frac{4}{3}\frac{m^2}{M_Z^2}\right.$$
$$\left.+\frac{1}{5}\left({}_2F_1\left(1,3,\frac{7}{2},\frac{\tau}{4}\right)+8{}_3F_2\left(1,1,3,2,\frac{7}{2},\frac{\tau}{4}\right)\right)\right) \quad (24)$$

$$F_2 = -\left(\frac{3}{2}-d\right)^2\left(\left(\frac{2}{3}\frac{m^2}{M_Z^2}-\frac{1}{3}\right)\frac{1+3s_w^2(q_d-q_u)+6s_w^4(q_u^2+q_d^2)}{1+2s_w^2(q_d-q_u)+4s_w^4(q_u^2+q_d^2)}{}_2F_1\left(2,2,\frac{5}{2},\frac{\tau}{4}\right)\right.$$
$$+\frac{1}{30}(8-\tau){}_2F_1\left(2,3,\frac{7}{2},\frac{\tau}{4}\right)+\frac{1}{35}{}_2F_1\left(2,4,\frac{9}{2},\frac{\tau}{4}\right)$$
$$\left.-\frac{2m^2}{3M_Z^2}\frac{1+3s_w^2(q_d-q_u)+6s_w^4(q_u^2+q_d^2)}{1+2s_w^2(q_d-q_u)+4s_w^4(q_u^2+q_d^2)}\right)$$
$$+\left(\frac{1}{12}+\frac{13}{6}d-\frac{4}{3}d^2\right){}_2F_1\left(1,1,\frac{5}{2},\frac{\tau}{4}\right) \quad (25)$$

and

$$F_3 = \frac{1}{6}\left(\left(d-\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_3F_2\left(1,d+\frac{1}{2},\frac{7}{2}-d,2,\frac{5}{2},\frac{\tau}{4}\right)+{}_2F_1\left(1,2,\frac{5}{2},\frac{\tau}{4}\right)\right)$$
$$+\left(\frac{3}{2}-d\right)^2\left(\left(-4\frac{m^2}{M_Z^2}+\frac{\tau}{4}\right){}_2F_1\left(1,2,\frac{5}{2},\frac{\tau}{4}\right)\right.$$
$$\left.-\frac{6}{5}{}_2F_1\left(1,3,\frac{7}{2},\frac{\tau}{4}\right)+4\frac{m^2}{M_Z^2}\right) \quad (26)$$

and

$$F_4 = \left(\frac{3}{2}-d\right)\left(\left(\frac{m^2}{M_Z^2}-\frac{1}{2}\right)\left({}_3F_2\left(1,d+\frac{1}{2},\frac{5}{2}-d,2,\frac{3}{2},\frac{\tau}{4}\right)+{}_2F_1\left(1,1,\frac{3}{2},\frac{\tau}{4}\right)\right)\right.$$
$$-2\frac{m^2}{M_Z^2}\frac{2}{\Gamma(5)}\left(\left(d+\frac{1}{2}\right)\left(\frac{5}{2}-d\right){}_3F_2\left(1,d+\frac{3}{2},\frac{7}{2}-d,3,\frac{5}{2},\frac{\tau}{4}\right)\right.$$
$$\left.\left.+2{}_2F_1\left(1,2,\frac{5}{2},\frac{\tau}{4})\right)\right)\right) \quad (27)$$

and

$$F_5 = \frac{2s_w^2(q_d - q_u + 2s_w^2(q_u^2 + q_d^2))}{1 + 2s_w^2(q_d - q_u) + 4s_w^4(q_u^2 + q_d^2)} \Bigg({}_3F_2(1, d - \frac{1}{2}, \frac{5}{2} - d, 1, \frac{3}{2}, \frac{\tau}{4}\Bigg) + {}_2F_1\left(1, 1, \frac{3}{2}, \frac{\tau}{4}\right) - 2\Bigg) \tag{28}$$

where $\tau = M_Z^2/m^2$ and ${}_2F_1$, ${}_3F_2$ and ${}_4F_3$ are the generalized hypergeometric functions. q_u, q_d are the electric charges in unit of e of the upper and lower component of the unparticle doublet $\mathcal{U}$. μ is the renormalization scale constant. In general, μ takes arbitrary values but since we are working with experimental data extracted at the LEP experiments, with momentum scale around the Z pole, we choose in the following study values of μ in the range $\mu \in [M_Z/2, 2M_Z]$.

3. Phenomenology

To find the region of parameter space of unparticle that is compatible with experimental limits, we must compare the unparticle contribution to the oblique parameters S and T to their experimental values deduced from electroweak precision measurements. Taking into account the discovery of the Higgs boson with mass $m_H = 125.18 \pm 0.16$, the fitted values of S and T, as reported in Ref. 8, are the following

$$\begin{aligned} \Delta S &= S - S_{\rm SM} = 0,05 \pm 0,11, \\ \Delta T &= T - T_{\rm SM} = 0,09 \pm 0,13. \end{aligned} \tag{29}$$

To illustrate the bounds on unparticles parameters from electroweak precision tests we present in Figs. 2 and 3 contour plots in the plane of (d, m) in the regions

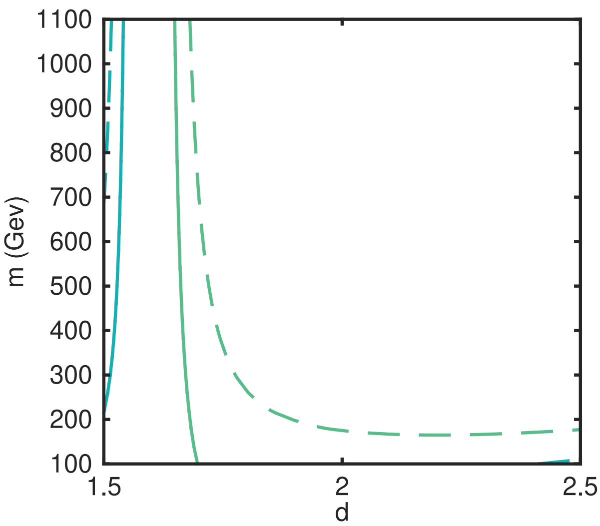

Fig. 2. Contour plots in the plane (d, m) for $S = 0,11$ on the right-hand side and $S = -0,11$ on the left-hand side, solid lines are contour plots for $\mu = 2M_Z$ and dashed lines are contour plots for $\mu = M_Z/2$.

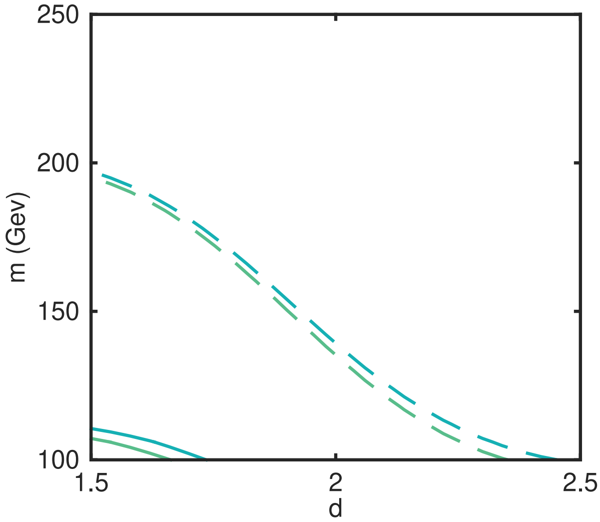

Fig. 3. Contour plots in the plane (d, m) for $T = 0,13$ represented by the upper solid and dashed lines and $T = -0,13$ represented by the lower solid and dashed lines, solid lines are contour plots for $\mu = M_Z$ and dashed lines are for $\mu = 2M_Z$.

$1,5 \leq d \leq 2,5$ and $100 \leq m \leq 1100$ for S and $100 \leq m \leq 250$ for T. In this study, we have chosen the values $q_u = -1$, $q_d = 0$ for the charges of the upper and lower components, respectively, of the unparticle multiplet. In Fig. 2, contour plots for experimental upper and lower bounds $S = 0,11$ and $S = -0,11$ are depicted for two choices of the renormalization scale μ. For $\mu = M_Z/2$, the solid line in the right-hand side represents $S = 0,11$ and the solid line in the left-hand side represents $S = -0,11$. For $\mu = 2M_Z$, the dashed line in the right represents $S = 0,11$ and the dashed line in the left represents $S = -0,11$. As can bee seen from this figure, for values of scale dimension $d \leq 1,7$, there is practically no constraints on the values of conformal breaking scale m but for values of $d \geq 1,7$ is restricted to values $m \leq 200$ Gev. The allowed region in the parameters space (d, m) becomes narrower as d increases. For $\mu = 2M_Z$, the scale dimension d must be inferior to $1,7$ to satisfies the experimental bounds. Figure 3 shows contour plots for the upper and lower experimental limits $T = 0,13$ (the upper solid and dashed lines) and $T = -0,13$ (the lower solid and dashed lines). The solid plots represent T for the renormalization scale value $\mu = M_Z$ and the dashed plots represent T for $\mu = 2M_Z$. The region between the two solid lines and the two dashed lines is consistent with measurements for the chosen renormalization scale value. It is clear from this figure that the oblique parameter T imposes a strong constraint on the allowed region of parameter space. For $\mu = 2M_Z$, values of the conformal breaking scale $m \geq 200$ are excluded in the range $1,5 \leq d \leq 2,5$. For $\mu = M_Z$, the allowed region is smaller. The allowed values of the scale dimension d shrinks to the range $1,5 \leq d \leq 1,7$ and $m \leq 110$.

Figures 2 and 3 are based on the bounds expressed by Eq. (29) in which S and T are taken as independent parameters. In reality, there is a correlation between these two observables expressed by the correlation coefficient $\rho = 0,9$.[8] Figure 4

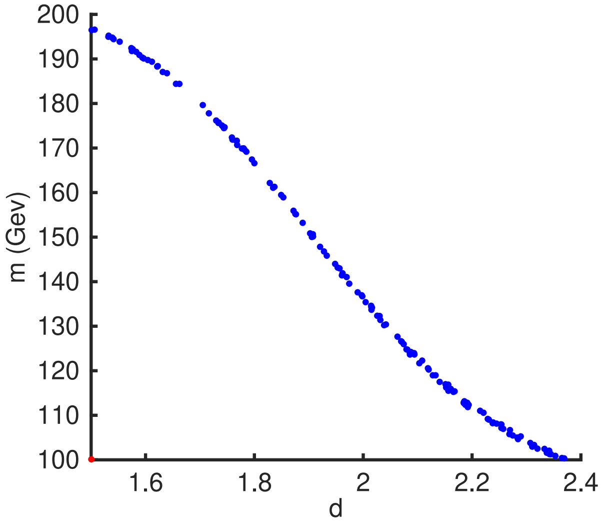

Fig. 4. (Color online) Scatter plot in the plane (d, m) wish show the region in parameters space compatible with the 1σ experimental bound.

shows scatter plots in the (d, m) plane compatible with 1σ experimental bounds of electroweak precision data in which the correlation coefficient ρ is taken into account. The blue dots represent scatter points for the renormalization scale value $2M_Z$. The red point represents the allowed region for $\mu = M_Z$. From this figure, we see that the allowed region is highly sensitive to the value of the renormalization scale in the chosen range. The infrared (IR) cutoff scale m is constrained to the interval $100 \leq m \leq 200$ but the scale dimension can take value up to 2.34 for $\mu = 2M_Z$. In general, the combined fitted results of S and T, expressed by Fig. 4, are compatible with the restrictions imposed by the oblique parameter T (Fig. 3) except that the allowed region gets smaller in the edges when d approaches $2, 4$ and the conformal breaking scale m approaches 200.

4. Conclusion

In this work, we have calculated the contribution of a gauged unparticle model, based on the electroweak group SU(2) × U(1), to the oblique parameters S and T. We have used the results of this calculation to construct the region in the parameters space (d, m) consistent with electroweak precision measurements represented by S and T. For different choices of the renormalization scale constant, we have found that the conformal breaking scale must lie in the interval $100 \leq m \leq 200$ Gev for $1, 5 \leq d \leq 2, 4$ in order to satisfy the experimental bounds. m is an IR cutoff scale introduced in the propagator (1) to parametrize our ignorance about the details of how scale invariance of the conformal sector (unparticle) is broken in the IR. The constraints on m mentioned above suggest that the unparticle physics is relevant only for experiments that probe energies $E \geq m \simeq 200$ Gev. Below this limit, the unparticle sector become an ordinary particle sector. Thus, the most promising probes of the unparticle effects must come from high-energy physics.

It is worth to mention that the unparticles can decay, just like normal particles. They can be regarded as a sum over several particle propagators, where the particles have a continuously distributed mass and a width related to the imaginary part of the loop correction as required by unitarity. Scalar unparticles with these interactions can be produced at colliders through gluon–gluon fusion, in the subprocesses $gg \to U$, $gg \to gU$. The unparticle can decay through the processes $U \to gg$ and $U \to \gamma\gamma$, leading to multijet events, or events with two photons plus jets. For the scale dimensions $d = 1.1$ and $d = 1.4$.[9] For larger values of the scale breaking m the decays are almost all prompt. For small m, more unparticles with a long lifetime can be produced, and we get a large number of monojets. This provides a new type of signal of unparticles. Note that if the unparticle is a singlet under SM gauge group transformations, there is a limited number of ways that the unparticles can couple to SM particles.[10] Another scenario is when the unparticles have electroweak quantum numbers. For example, unquarks can decay into ordinary quarks and will have a resemblance to a fourth family. It is very important to mention that unparticles can decay even if they are singlets under SM gauge group transformations (they do not carry SM quantum numbers).[11] If we consider unparticles as a fourth-generation quarks, the collider bounds on masses, precision observables, and the renormalization flow of coupling are equivalent to imposing constraints on gauged unparticles parameters, which depend on the process and type of unparticles (scalar, vector, spinor, or tensor) under consideration. The analysis of electroweak precision tests imposes severe bounds on the involved parameter space, particularly the quark mixing between the third and fourth family and the possible mass differences within the new quark and lepton doublets. Constraints on the masses of the fourth-family fermions are obtained from their contributions to the electroweak correction parameters S and T. The CMS Collaboration put a lower bound on the mass of fourth-generation up-type quark of about 450 GeV and exclude fourth-generation down-type quark in the mass region 255–361 GeV at 95% C.L.[12]

To see the effect of the experimental results on the unparticles parameters space when decaying or interacting with SM particles, let us consider the work of Ref. 7 where it was shown that the scalar- and spinor-colored unparticle loop contributions have an important impact on Higgs phenomenology at the LHC and can explain the excess in $h \to \gamma\gamma$ observed by ATLAS experiment. In fact in the scalar case, an enhancement in the Higgs diphoton decay rate requires a negative coupling $\lambda h U_s$ and a large electrical charge to restore the naturalness and vacuum stability, while in the spinor case an enhancement can be obtained by either negative or positive coupling to the Higgs boson depending on the scale dimension d_f due to the flipping of the sign of the spin-1/2 contributions. In both cases, a significant enhancement of $h \to \gamma\gamma$ selects a very special region of the unparticle parameters. The present data of ATLAS in diphoton decay rate of SM-like Higgs boson around 125 GeV serve to constrain the unparticle parameter model. Concerning the uncolored unparticles, both scalar and tensorial interactions to SM fields can lead to sizable observable effects in the invariant mass distributions of dilepton pairs at hadron colliders in the large invariant mass region.[13] Missing energy from the unparticle at hadronic collisions are explored. The complex phase in the unparticle propagator that can give rise to interesting interference effects between an unparticle exchange diagram and the SM amplitudes are found sensitively depending not only on the scale dimension but also on the spin of the unparticle. Furthermore the possible effects of unparticles through photon–photon scattering, rare annihilation type B decays, top quark rare decays, and comparison with experimental data put severe constraints on the unparticles parameter space. As a concrete example for the triangle exchange of fermionic unparticles to saturate the upper bound of the electron, muon and neutron electromagnetic dipole moments, one has to have $\Lambda_U = 1$ TeV (energy scale at which unparticles emerge), $m = 200$ GeV, $d \in [1.5, 2]$.[14] In the electroweak gauge boson W scattering and since the vector unparticles propagator depends on the scale dimension d measuring the angular distribution of the W boson, one can determine the scale dimension d. For the scalar signal at the LHC,[15] A detailed study of certain processes within the unparticles scenario $pp \to 4\gamma \dots pp \to 2\gamma 2g \dots pp \to 2\gamma 2l$, $pp \to 4e \dots pp \to 4\mu \dots pp \to 2e2\mu$ at $\sqrt{s} = 14$ TeV is carried out. Using basic selection cuts and analyze various distributions to discriminate the signals over the SM background. Using the experimental values, limits on the uncolored scalar unparticles parameters d and Λ_U are set. The bound on Λ_U can get as large as 1 TeV for small d values, but it is smaller for larger values.

Finally, we conclude that the unparticles (gauged or ungauged) decays and interactions with the SM particles are very important and lead to sizable effects. Imposing the collider and/or electroweak precision data tests, which are in general complementary will affect more the parameter space region like (Λ_U, d) etc., depending on the type of unparticles (scalars, vectors ...) and the process under consideration. In the present case if we consider the unparticles effect like the one of the fourth generation of quarks, we believe that the collider bounds on the fourth generation of fermions will impose stringent constraints on the other gauged fermionic unparticles parameters like the unparticle SM charge Q_U and Yukawa coupling λ_U, and Λ_U.

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References

1. H. Georgi, *Phys. Rev. Lett.* **98**, 221601 (2007).
2. G. Cacciapaglia, G. Marandella and J. Terning, *JHEP* **01**, 070 (2008).
3. M. E. Peskin and T. Takeuchi, *Phys. Rev. D* **46**, 381 (1992).
4. J. Terning, *Phys. Rev. D* **44**, 887 (1991).
5. J. C. Ward, *Phys. Rev.* **78**, 182 (1950).

6. R. Basu, D. Choudhury and H. Mani, *Eur. Phys. J. C* **61**, 461 (2009).
7. I. Aliane, N. Mebarki and Y. Delenda, *Phys. Lett. B* **728**, 549 (2014).
8. M. Baak and R. Kogler, arXiv:1306.0571.
9. A. Rajaraman, *Phys. Lett. B* **671**, 411 (2009).
10. S. L. Chen and X. G. He, *Phys. Rev. D* **76**, 091702 (2007).
11. B. Holdom, *JHEP*, **2006**, 076 (2006).
12. M. A. Schmidt and A. Y. Smirnov, *Nucl. Phys. B* **857**, 1 (2012).
13. P. Mathews and V. Ravindran, *Phys. Lett. B* **657**, 198 (2007).
14. Y. Liao, *Phys. Lett. B* **665**, 356 (2008).
15. N. Greiner, *Phys. Lett. B* **653**, 75 (2007).

Appendix D

conference paper

Muon Anomalous Magnetic Moment in the Left-Right Symmetric Model

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Muon Anomalous Magnetic Moment in the Left-Right Symmetric Model

J H Taibi[1] **and N Mebarki**
Laboratoire de Physique MathématiqueetSubatomique,
Mentouri University, Constantine, Algeria

E-mail: taibi.hamza@hotmail.com

Abstract. The measurement of muon anomalous magnetic moment provides a test of the standard model and of the physics that lies beyond it. Currently, there is a deviation of 2.6σbetween the standard model prediction and the experimental results. In this work, the contribution of heavy gauge bosons from the left right symmetric model (LRSM) is calculated. We find that the LRSM can give a relatively small but non-negligible extra weak contribution to the muon anomalous magnetic moment and can reduce the deviation of Δa_μ from *2.6σ* for the SM to *2.5σ* for the LRSM model.

Introduction

1. Introduction

The SM has been so far in excellent agreement with experiment. However, some problems do not have any explanation in its minimal version. Among them, the hierarchy problem (the stability of the higgs mass under radiative corrections) [1,2], neutrino masses [3] and the anomalous magnetic moment of the muon [4], which is the subject of this work.
The left right symmetric model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is the most natural extension of the standard model. It can explains parity violation at low energies [5,6] and provide a seesaw mechanism that give masses to neutrinos in a natural way.
In our work, we use the LRSM to explain the anomalous magnetic moment of the muon (AMM). The study of AMM represents a very sensitive test of the SM at the quantum loop level and permits the investigation of physics that lie beyond it. The magnetic moment is defined as $\mu = g(e/2m)s$, where g is the gyromagnetic ratio. The deviation of the magnetic moment from the value of the point-like Dirac particle $(g = 2)$ is induced by the interactions of leptons with virtual particles which couple to electromagnetic field. Whereas the electron anomaly provides the most precise measurement of the fine structure constant, the muon anomaly is more sensitive to virtual gauge bosons.In this paper, we investigate the effects of the left right symmetric model on anomalous magnetic moment of the muon. We consider all possible contributions from extra gauge bosons at the one loop level. Our purpose is to get a better interpretation of the experimental results of the muon anomaly. In section 2, we give a short review of the LRSM, its theoretical basis and the structure of the gauge sector. In section 3, we give the calculation of muon g-2 in the LRSM. In section 4, we give a numerical estimation of the

[1] To whom any correspondence should be addressed.

value of the anomalous magnetic moment of the muon. Finally, a short summary and conclusion are given.

2. Review of the left right symmetric model

The main motivation for the left right symmetric model is the treatment of the right-handed particles and their interaction on an equal footing with the left ones. In this model the weak interaction is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $B-L$ stand for the difference in baryon and lepton numbers. The symmetry breaking (SB) of the LRSM is achieved in two steps via the vacuum expectation value (VEV) of scalar triplets and multiples.

$$\Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}_{L,R}, \quad \phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{1}$$

The symmetry-breaking scheme is as follows

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\Delta_R} SU(2)_L \times U(1)_Y \tag{2}$$

At this stage the right scalar develop a vev $\langle \Delta_R \rangle = V_R$ and break the left right symmetry to the electroweak symmetry giving masses to heavy electroweak gauge bosons. After the first stage of SM, the kinetic term of the higgs sector become

$$Tr|D_\mu \Delta_R|^2 = \frac{g_R^2 V_R^2}{2} W_R^{\mu-} W_{R\mu}^+ + \frac{V_R^2}{2}(g_R W_R^{\mu 3} - g_{B-L} V^\mu)(g_R W_R^{\mu 3} - g_{B-L} V^\mu) \tag{3}$$

The physical heavy field Z_R, and the $U(1)_Y$ gauge field B_μ are derived by appling unitry transformation characterized by the mixing angle φ

$$\begin{pmatrix} Z_{R\mu} \\ B_\mu \end{pmatrix} = \begin{pmatrix} cos\,(\varphi) & -sin\,(\varphi) \\ sin\,(\varphi) & cos\,(\varphi) \end{pmatrix} \begin{pmatrix} W_{R\mu}^3 \\ V_\mu \end{pmatrix} \tag{4}$$

Where φ is defined by

$$cos(\varphi) = \frac{g_R}{\sqrt{g_R^2 + g_{B-L}^2}}, \quad sin(\varphi) = \frac{g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}} \tag{5}$$

In the second stage of symmetry breaking, the other higgs fields ϕ and Δ_L get vev and give masses to the SM gauge bosons W_L and Z_L

$$\langle \phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ V_L & 0 \end{pmatrix} \tag{6}$$

The gauge bosons masses are given by

$$M_A = 0, \tag{7}$$

$$M_{Z_L}^2 = \frac{g_R^2}{2} \frac{1}{cos^2(\theta_W)} (\kappa_1^2 + \kappa_2^2 + 4V_L^2), \tag{8}$$

$$M_{Z_R}^2 = 2(g_R^2 + g_{B-L}^2) V_R^2 \tag{9}$$

$$M^2_{W_L^{\mp}} = \frac{g_R^2}{2}(\kappa_1^2 + \kappa_2^2 + 4V_L^2) \quad (10)$$

$$M^2_{W_R^{\mp}} = 2g_R^2 V_R^2 \quad (11)$$

3. Calculation of a_μ in the left right symmetric model

3.1. Values of a_μ in the SM

The muon anomaly in the SM is the summation of three contributions

$$a_\mu^{SM} = a_{\mu,SM}^{QED} + a_{\mu,SM}^{Weak} + a_{\mu,SM}^{Had} \quad (12)$$

These contributions have been determined precisely in previews works. The QED contribution is the dominant one, and it has been calculated up to the fourth order α^4. The weak contribution has been calculated up to 2 and 3 loop level and it has not changed much in the last years.
We present below the best results of the muon anomaly calculation in the SM [7,8,9]

$$a_{\mu,SM}^{QED} = 11658471.958(0.143) \times 10^{-10} \quad (13)$$

$$a_{\mu,SM}^{Weak} = 15.4(0.2) \times 10^{-10} \quad (14)$$

$$a_{\mu,SM}^{Had} = 697.2(5.9) \times 10^{-10} \quad (15)$$

The total SM value for a_μ is

$$a_\mu^{SM} = 11659184.56(5.9) \times 10^{-10} \quad (16)$$

and the present experimental value for a_μ is

$$a_\mu^{Exp} = 11659208.56(6) \times 10^{-10} \quad (17)$$

thus, the deviation of the experimental value of the anomalous magnetic moment of the muon from the SM prediction is

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM} = 23.4(9.0) \times 10^{-10} \quad (18)$$

3.2.Calculation of a_μ in the LRSM

The LRSM contribution to the muon anomaly is calculated using the diagrams of fig 1. In our calculation, we use the t'Hooft Feynman gauge for the propagator of gauge bosons. The total amplitude for the diagrams of fig1 can be written as

$$M_{LR} = -e\varepsilon_\mu(q)\bar{u}(p')\Gamma^\mu u(p) \quad (19)$$

Where q is the four-momentum of the photon, p and p' are momentum of the incoming and outgoing muon respectively, Γ^μ is the vertex function which has the general Lorentz structure

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_\mu} + F_3(q^2)\frac{i\sigma^{\mu\nu}q_\nu\gamma_5}{2m_\mu} \quad (20)$$

Because $F_1(q^2)$,$F_2(q^2)$ and $F_3(q^2)$ are related to the electric charge, the anomalous magnetic moment and the electric dipole moment respectively, we must calculate only the form factor $F_2(q^2)$. The LRSM contribution to the muon anomaly is derived as

$$a_{LR}(1loop) = \frac{g-2}{2} = F_2(0) \tag{21}$$

In our calculation we find that the most important contribution to muon anomaly came from the extras gauge bosons W_R,Z_R related to the gauge group $SU(2)_R$.For W_R, we find the following result for the vertex function

$$\Gamma^\mu = \frac{ig^2 e m_\mu}{64\pi^2 M_W^2} \times \frac{7}{6}(p + p')^\mu \tag{22}$$

using the Gordon identity $(p + p')_\mu = 2m\gamma_\mu - i\sigma^{\mu\nu}q_\mu$ and the definition of the AMM (eq(21)), we find the following result

$$a_{W_R}^\mu \cong \frac{\alpha}{16\pi sin^2(\theta_W)}\left(\frac{m_\mu}{M_{W_R}}\right)^2 \frac{7}{3} + O\left(\left(\frac{m_\mu}{M_{W_R}}\right)^4\right) \tag{23}$$

where α is the fine structure constant.

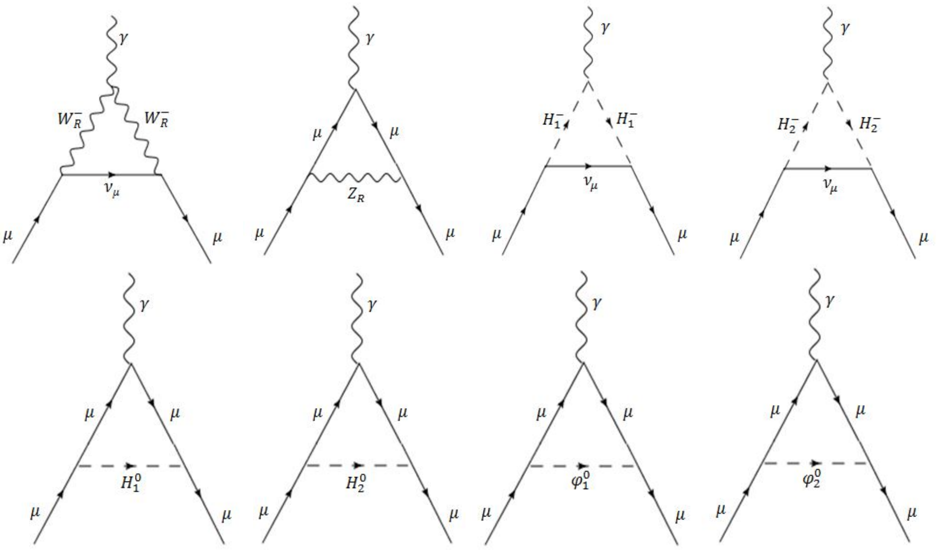

Figure 1. The electroweak one loop Feynman diagrams of the muon anomalous magnetic moment in the left right symmetric model

In the same manner, we calculate the heavy neutral gauge boson (Z_R) correction to the vertex function, and we deduce the corresponding muon anomaly

$$a_{Z_R}^\mu = -\frac{\alpha m_\mu^2}{12\pi M_{Z_R}^2}\frac{\left(1-tan^2(\theta)\left(1+tan^2(\theta)\right)\right)}{sin^2(\theta)(1-tan^2(\theta))} \tag{24}$$

The calculation of the muon anomaly corresponding to the charged higgs, represented by diagrams three and four in Fig 1, show that its contribution is negligible compared to the W and Z contribution.

For the scalar neutral higgs H_1^0, H_2^0 and the neutrals pseadoscalar higgs φ_1^0 and φ_2^0, we get the following results

$$a_{H_1^0}^{\mu} \approx \frac{\alpha}{8\pi sin^2(\theta)} \frac{tan^2(\beta)\left(1+tan^2(\beta)\right)}{(1-tan^2(\beta))^2} \left(\frac{m_\mu}{M_{W_L}}\right)^2 \left(\frac{m_\mu}{M_{H_1^0}}\right)^2 ln\left(\frac{M_{H_1^0}^2}{m_\mu^2}\right) \tag{25}$$

$$a_{H_2^0}^{\mu} \approx \frac{\alpha}{8\pi sin^2(\theta)} \frac{\left(1+tan^2(\beta)\right)}{(1-tan^2(\beta))^2} \left(\frac{m_\mu}{M_{W_L}}\right)^2 \left(\frac{m_\mu}{M_{H_2^0}}\right)^2 ln\left(\frac{M_{H_1^0}^2}{m_\mu^2}\right) \tag{26}$$

$$a_{\varphi_1^0}^{\mu} = \frac{\alpha}{8\pi sin^2(\theta)} \frac{tan^2(\beta)\left(1+tan^2(\beta)\right)}{(1-tan^2(\beta))^2} \left(\frac{m_\mu}{M_{W_L}}\right)^2 \int_0^1 dx \frac{x^3}{x^2+(1-x)\frac{M_{\varphi_1^0}^2}{m_\mu^2}} \tag{27}$$

$$a_{\varphi_2^0}^{\mu} = \frac{\alpha}{8\pi sin^2(\theta)} \frac{\left(1+tan^2(\beta)\right)}{(1-tan^2(\beta))^2} \left(\frac{m_\mu}{M_{W_L}}\right)^2 \int_0^1 dx \frac{x^3}{x^2+(1-x)\frac{M_{\varphi_2^0}^2}{m_\mu^2}} \tag{28}$$

Where β is a free parameter of the LRSM defined as

$$tan(\beta) = \frac{\kappa_1}{\kappa_2} \tag{29}$$

3.3. Numerical results

To get an estimate of the value of muon anomalous magnetic moment in the LRSM, we use the following values for the LRSM parameters: $\alpha = \frac{1}{137}$, $M_{Z_L} = 90Gev, M_{W_R} = 1Tev$, $M_{Z_R} = Tev, M_{H_1^0} = M_{H_2^0} = M_{\varphi_1^0} = M_{\varphi_2^0} = 5Tev, tan(\beta) = 10, sin^2(\theta) = 0.223$. After the summation of all contributions, we get the final result

$$a_{LR}^{\mu} = 0.137 * 10^{-10} \tag{30}$$

so, the deviation of the experimental value of the AMM of the muon from the SM prediction is reduced in the LRSM to

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{LR} = 23.26(0.9) \times 10^{-10} \tag{31}$$

4. Conclusions and Summary

The LRSM is an alternative candidate of new physics beyond the SM which can explain parity violation at low energies. The LRSM predicts new particles, such as heavy gauge bosons (W_R, Z_R), and heavy charged and neutral higgs *($H_{1,2}^-, H_{1,2}^0, \varphi_{1,2}^0$)*. In this work, we have calculated the muon anomalous magnetic moment at the one loop level in the LRSM. We find that the LRSM electroweak contribution can reduce slightly the deviation of the theoretical prediction of muon AMM from the experimental result. The total contribution of LRSM to muon *g-2* is about 0.137×10^{-10}, so the muon anomaly decreases from $\Delta a_\mu = 23.4 \times 10^{-10}$in the SM to $\Delta a_\mu = 23.26 \times 10^{-10}$ in the LRSM, which is 0.6 % smaller. We conclude that the LRSM gives a small but non-negleagble extra contribution to muon *g-2*, and reduce the deviation Δa_μ from 2.6σ in the SM to 2.5σ in the LRSM.

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References

[1] Casalbuoni R, Deandrea A, and Oertel M 2004*JHEP* 0402
[2] t'Hooft G 1997*Cambridge University Press, Cambridge*
[3] Mohapatra R N and Pal P B 2004*World, Scientific, Third Edition, Lecture Notes in Physics,***72**
[4] Jegerlehner F and Nyffeler A 2009*Phys. Rept.***477** 1
[5] Senjanovic G, and Mohapatra R N 1975 *Phys. Rev.* D **12** 1502
[6] Pati J C and Salam A1974*Phys. Rev.* D**l0** 275
[7] Bennett G W et al. Muon (g−2) Collaboration 2004 *Phys.Rev.Lett.***92** 161802
[8] Czarnecki A, Marciano W A and Vainshtein A 2003 *Phys.Rev.* D **67** 073006
[9] Melnikov K and Vainshtein A 2004 *Phys. Rev.* D **70**113006

Bibliography

[1] ATLAS Collaboration. Observation of a new particle in the search for the standard model higgs boson with the atlas detector at the lhc. *Physics Letters B*, 716(1):1–29, 2012.

[2] CMS Collaboration. [2] cms collaboration, observation of a new boson with mass near 125 gev in pp collisions at s=7 and 8 tev. *JHEP*, 06:081, 2013.

[3] Howard Georgi. Unparticle physics. *Physical Review Letters*, 98(22):221601, 2007.

[4] Tatsuru Kikuchi and Nobuchika Okada. Unparticle dark matter. *Physics Letters B*, 665(4):186–189, 2008.

[5] Giacomo Cacciapaglia, Guido Marandella, and John Terning. Colored Unparticles. *JHEP*, 01:070, 2008.

[6] Babak Abi, T Albahri, S Al-Kilani, D Allspach, LP Alonzi, A Anastasi, A Anisenkov, F Azfar, K Badgley, S Baeßler, et al. Measurement of the positive muon anomalous magnetic moment to 0.46 ppm. *Physical Review Letters*, 126(14):141801, 2021.

[7] Anirban Kundu and Probir Roy. A general treatment of oblique parameters. *International Journal of Modern Physics A*, 12(08):1511–1529, 1997.

[8] G. A. Kozlov. Comments on gauge unparticles. *Phys. Part. Nucl.*, 41:957–961, 2010.

[9] Yi Liao. Some issues in a gauge model of unparticles. *The European Physical Journal C*, 60(1):125–134, 2009.

[10] Enrico Fermi. Quantum theory of radiation. *Reviews of modern physics*, 4(1):87, 1932.

[11] Sheldon L Glashow. The renormalizability of vector meson interactions. *Nuclear Physics*, 10:107–117, 1959.

[12] J. C. Salam, A.; Ward. Weak and electromagnetic interactions. *Nuovo Cimento*, 11:568–577, 1959.

[13] Steven Weinberg. A model of leptons. *Physical review letters*, 19(21):1264, 1967.

[14] François Englert and Robert Brout. Broken symmetry and the mass of gauge vector mesons. *Physical Review Letters*, 13(9):321, 1964.

[15] Peter W Higgs. Broken symmetries and the masses of gauge bosons. *Physical Review Letters*, 13(16):508, 1964.

[16] Jeffrey Goldstone. Field theories with superconductor solutions. *Il Nuovo Cimento (1955-1965)*, 19(1):154–164, 1961.

[17] Jeffrey Goldstone, Abdus Salam, and Steven Weinberg. Broken symmetries. *Physical Review*, 127(3):965, 1962.

[18] Michael E Peskin and Tatsu Takeuchi. New constraint on a strongly interacting higgs sector. *Physical Review Letters*, 65(8):964, 1990.

[19] Michael E Peskin and Tatsu Takeuchi. Estimation of oblique electroweak corrections. *Physical Review D*, 46(1):381, 1992.

[20] Max Baak and Roman Kogler. The global electroweak standard model fit after the higgs discovery. *arXiv preprint arXiv:1306.0571*, 2013.

[21] Yasuhiro Okada, Masahiro Yamaguchi, and Tsutomu Yanagida. Upper bound of the lightest higgs boson mass in the minimal supersymmetric standard model. *Progress of Theoretical Physics*, 85(1):1–5, 1991.

[22] F. Zwicky. Die rotverschiebung von extragalaktischen nebeln. *Helvetica Physica Acta*, 6:110–127, 1933.

[23] Adam G Riess, Alexei V Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M Garnavich, Ron L Gilliland, Craig J Hogan, Saurabh Jha, Robert P Kirshner, et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, 116(3):1009, 1998.

[24] Yoshiyuki Fukuda, T Hayakawa, E Ichihara, K Inoue, K Ishihara, Hirokazu Ishino, Y Itow, T Kajita, J Kameda, S Kasuga, et al. Evidence for oscillation of atmospheric neutrinos. *Physical Review Letters*, 81(8):1562, 1998.

[25] G Mack and Abdus Salam. Finite-component field representations of the conformal group. In *Selected Papers Of Abdus Salam: (With Commentary)*, pages 255–283. World Scientific, 1994.

[26] Sergio Ferrara, Raoul Gatto, and A Grillo. Positivity restriction on anomalous dimensions. *Physical Review D*, 9(12):3564, 1974.

[27] Kenneth G Wilson. Renormalization group and critical phenomena. i. renormalization group and the kadanoff scaling picture. *Physical review B*, 4(9):3174, 1971.

[28] Joseph Polchinski. Renormalization and effective lagrangians. *Nuclear Physics B*, 231(2):269–295, 1984.

[29] Tom Banks and Alexander Zaks. On the phase structure of vector-like gauge theories with massless fermions. *Nuclear Physics B*, 196(2):189–204, 1982.

[30] Gunnar Kallen. On the definition of the Renormalization Constants in Quantum Electrodynamics. *Helv. Phys. Acta*, 25(4):417, 1952.

[31] H. Lehmann. On the Properties of propagation functions and renormalization contants of quantized fields. *Nuovo Cim.*, 11:342–357, 1954.

[32] M. A. Stephanov. Deconstruction of Unparticles. *Phys. Rev. D*, 76:035008, 2007.

[33] Patrick J. Fox, Arvind Rajaraman, and Yuri Shirman. Bounds on Unparticles from the Higgs Sector. *Phys. Rev. D*, 76:075004, 2007.

[34] Antonio Delgado, Jose R. Espinosa, and Mariano Quiros. Unparticles Higgs Interplay. *JHEP*, 10:094, 2007.

[35] Kingman Cheung, Wai-Yee Keung, and Tzu-Chiang Yuan. Collider Phenomenology of Unparticle Physics. *Phys. Rev. D*, 76:055003, 2007.

[36] Kingman Cheung, Wai-Yee Keung, and Tzu-Chiang Yuan. Phenomenology of unparticle physics. *Int. J. Mod. Phys. A*, 24:3508–3514, 2009.

[37] Juan Maldacena. The large-n limit of superconformal field theories and supergravity. *International journal of theoretical physics*, 38(4):1113–1133, 1999.

[38] Martin Ammon and Johanna Erdmenger. *Gauge/gravity duality: Foundations and applications.* Cambridge University Press, Cambridge, 4 2015.

[39] Giacomo Cacciapaglia, Guido Marandella, and John Terning. The AdS/CFT/Unparticle Correspondence. *JHEP*, 02:049, 2009.

[40] Howard Georgi and Yevgeny Kats. Unparticle self-interactions. *Journal of High Energy Physics*, 2010(2):1–62, 2010.

[41] John Terning. Gauging nonlocal lagrangians. *Physical Review D*, 44(3):887, 1991.

[42] B Holdom, J Terning, and K Verbeek. Chiral lagrangian from quarks with dynamical mass. *Physics Letters B*, 245(3-4):612–618, 1990.

[43] Anton Ilderton. Unparticle actions and gauge invariance. *Physical Review D*, 79(2):025014, 2009.

[44] Patricio Gaete and Euro Spallucci. Un-particle effective action. *Physics Letters B*, 661(4):319–324, 2008.

[45] Kenneth G Wilson. Confinement of quarks. *Physical review D*, 10(8):2445, 1974.

[46] Stanley Mandelstam. Quantum electrodynamics without potentials. *Annals Phys.*, 19:1–24, 1962.

[47] John Clive Ward. An identity in quantum electrodynamics. *Physical Review*, 78(2):182, 1950.

[48] Yasushi Takahashi. On the generalized ward identity. *Il Nuovo Cimento (1955-1965)*, 6(2):371–375, 1957.

[49] Rahul Basu, Debajyoti Choudhury, and HS Mani. Fermionic unparticles, gauge interactions and the β function. *The European Physical Journal C*, 61(3):461–470, 2009.

[50] Arvind Rajaraman. On the decay of unparticles. *Physics Letters B*, 671(3):411–414, 2009.

[51] Shao-Long Chen and Xiao-Gang He. Interactions of unparticles with standard model particles. *Physical Review D*, 76(9):091702, 2007.

[52] Bob Holdom. The discovery of the fourth family at the lhc: what if? *Journal of High Energy Physics*, 2006(08):076, 2006.

[53] Michael A Schmidt and Alexei Yu Smirnov. Neutrino masses and a fourth generation of fermions. *Nuclear Physics B*, 857(1):1–27, 2012.

[54] Idir Aliane, Noureddine Mebarki, and Yazid Delenda. Enhancement of h $\gamma\gamma$ via spin-0 and spin-1/2 charged unparticle loops. *Physics Letters B*, 728:549–553, 2014.

[55] Prakash Mathews and V Ravindran. Unparticle physics at hadron collider via dilepton production. *Physics Letters B*, 657(4-5):198–206, 2007.

[56] MC Kumar, Prakash Mathews, V Ravindran, and Anurag Tripathi. Unparticle physics in diphoton production at the cern lhc. *Physical Review D*, 77(5):055013, 2008.

[57] Yi Liao. Electromagnetic dipole moments and radiative decays of particles from exchange of fermionic unparticles. *Physics Letters B*, 665(5):356–360, 2008.

[58] Nicolas Greiner. Constraints on unparticle physics in electroweak gauge boson scattering. *Physics Letters B*, 653(1):75–80, 2007.

[59] Yi Liao. Effects of unparticles on running of gauge couplings. *The European Physical Journal C*, 55(3):483–488, 2008.

[60] Ilia Gogoladze, Nobuchika Okada, and Qaisar Shafi. Unparticle physics and gauge coupling unification. *Physics Letters B*, 659(1-2):357–360, 2008.

[61] John Ellis, S Kelley, and Dimitri V Nanopoulos. Precision lep data, supersymmetric guts and string unification. *Physics Letters B*, 249(3-4):441–448, 1990.

[62] Howard Georgi, Helen R Quinn, and Steven Weinberg. Hierarchy of interactions in unified gauge theories. In *Origin Of Symmetries*, pages 334–337. World Scientific, 1991.

[63] JC Pati and A Salam. Lepton number as the fourth color 1974. *Phys. Rev. D*, 10:275.

[64] Rabindra N Mohapatra and Jogesh C Pati. " natural" left-right symmetry. *Physical Review D*, 11(9):2558, 1975.

[65] Gt Senjanovic and Rabindra N Mohapatra. Exact left-right symmetry and spontaneous violation of parity. *Physical Review D*, 12(5):1502, 1975.

[66] RH Mohapatra. Left-right symmetric models of weak interactions: a review. *Quarks, leptons, and beyond*, 1983.

[67] Andrzej Czarnecki and William J Marciano. Muon anomalous magnetic moment: A harbinger for "new physics". *Physical Review D*, 64(1):013014, 2001.

[68] Joydeep Chakrabortty, Janusz Gluza, Tomasz Jeliński, and Tripurari Srivastava. Theoretical constraints on masses of heavy particles in left-right symmetric models. *Physics Letters B*, 759:361–368, 2016.

Abstract

Though very successful at describing a wide variety of particle physics phenomenons, the standard model leaves some proprieties of nature unexplained. This fact lead physicists to search for new solutions based on extended symmetries or extra degree of freedoms. In this thesis we focus on two of the new physics proposal, the unparticle model and the left right symmetric model. We begin by presenting the theoretical framework upon which unparticle are founded, conformal field theory. Then, we describe briefly the original formulation of the unparticle model by Georgi and subsequent works relevant to our study. Next, we construct a gauged unparticle model charged under the standard model electroweak group $SU(2) \times U(1)$. Any new physics model must be compatible with the standard model predictions. In this regard the electroweak precision tests represent a powerful tool to check the compatibility of new physics model with experimental data. For the gauged unparticle model we calculate the oblique parameters S and T that parametrize the contribution of new physics states to electroweak observables. We then use the bound for these parameters extracted from LEP and other experiments to constrain the parameters space of unparticles. Finally, we consider the effects of left right symmetric model spectrum on the muon anomalous magnetic moment and use it to decrease the deviation between the SM predictions and experimental observations . A paper describing part of the results of this dissertation has been published in the Journal Modern Physics Letters A, entitled: "Constraints on electroweak gauged unparticle model from the oblique parameters S and T".

Keywords: unparticles , gauged model, oblique parameters, left right symmetry, muon anomaly

Résumé

Le modèle standard est une théorie très efficace pour décrire une grande variété de phénomènes de physique des particules, mais il n'éxplique pas certaines propriétés de la nature. Ce fait a conduit les physiciens à rechercher une nouvelle solution basée sur des symétries étendues ou des degrés de libertés supplémentaires. Dans cette thèse, nous nous concentrons sur deux propositions du physique au dela du modéle standard , les unparticles et le modele symétrique gauche droite. Nous commençons par présenter le cadre théorique sur lequel les unparticles sont fondées, la théorie des champs conformes. Ensuite, nous décrivons brièvement la formulation originale du modèle unparticle due à Georgi et les travaux ultérieurs liés à notre étude. Ensuite, nous construisons un modèle de gauge pour les unparticle, précisément nous considérons des unparticles chargés sous le groupe du modèle standard électrofaible $SU(2) \times U(1)$. Tout nouveau modèle de physique doit être compatible avec les prédictions du modèle standard. À cet égard, les tests de précisions électrofaibles représentent un outil efficace pour vérifier la compatibilité du nouveau modèle avec les données expérimentales. Pour le modèle unparticle chargé, nous calculons les paramètres obliques S et T qui paramètrisent la contribution des nouveaux états physiques aux observables électrofaibles. Nous utilisons ensuite les bornes de ces paramètres extraits des expériences LEP et autres pour construire l'espace des paramètres des unparticle compatible avec l'experience. Enfin, nous considérons les effets des spectres du modèle gauche droite sur le moment magnétique anomale du muon et nous l'utilisons pour diminuer la déviations entre la prédiction du modèle standard et les observations expérimentales. Un article décrivant une partie des résultats de cette thèse a été publié dans le Journal Modern Physics Letters A, intitulés: "Constraints on electroweak gauged unparticle model from the oblique parameters S and T".

Mots-clés: unparticles, modèle de gauge, paramètres obliques, symétrie droite gauche, anomalie du muon

ملخص

النموذج العياري حقق نجاحا كبيرا في وصف الكثير من الظواهر المرتبطة بفيزياء الجسيمات الا انه فشل في تفسير بعض الخصائص الطبيعية. هذا الواقع حفز الفيزيائيين على البحث عن حلول جديدة مبنية على تناظرات موسعة أو درجات حرية اضافية. في هذه الأطروحة نركز على اثنين من هذه الحلول المقترحة نموذج الاجسيمات و نموذج التناظر يمين يسار. نبدأ بتقديم الاطار النظري الذي بني عليه نموذج الاجسيمات وهي نظرية الحقول المتوافقة. بعدها نصف باختصار الصياغة الاصلية لنموذج الاجسيمات، اللتي ترجع للفيزيائي جيورجي، والاعمال اللاحقة المرتبطة بموضوعنا. بعدها نقوم بصياغة نموذج للاجسيمات التي تحمل شحنات الزمرة العيارية $SU(2) * U(1)$. كل اقتراح فيزياء جديدة يجب ان يكون متوافقا مع تنبؤات النموذج العياري، في هذا السياق تمثل اختبارات الدقة الكهروضعيفة وسيلة فعالة للتحقق من توافق نموذج فيزياء جديدة مع المعطيات التجريبية. بالنسبة للاجسيمات التي تحمل شحنات الزمرة العيارية نقوم بحساب المعاملات المائلة S و T التي تلخص مساهمة حقول الفيزياء الجديدة في المرصودات الكهروضعيفة. بعدها نقوم باستخدام القيود على قيم المعاملات المائلة المستخرجة من تجارب LEP وغيرها لحصر فضاء المعاملات للاجسيمات. أخيرا، ندرس تأثير طيف نموذج التناظر يمين يسار على العزم المغناطيسي غير الطبيعي للميون ونستخدمه لانقاص الفرق بين تنبآت النموذج العياري و الملاحظات التجريبية

كلمات مفاتيح : الاجسيمات, نموذج جوجي, المعاملات المائلة، تناظر يمين يسار، انحراف الميون