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The numerical study of the convective heat transfer between two horizontal eccentric cylinders

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Abstract

Convective heat transfer in eccentric horizontal annuli situated between two cylinders with isothermal wall conditions are studied numerically using two-dimensional finite-difference and finite-volumes models. The annular space is filled by a Newtonian and incompressible fluid. The effects of various parameters such as the radius ratio of the annulus, the eccentricity of the annulus, the Grashof number are investigated at the Prandtl number of 0.701. By using the approximation of Boussinesq and the vorticity-stream function formulation, the flow is modeled by the differential equations with the derivative partial: the equations of continuity and the momentum are expressed in a frame of reference known as "bicylindrical", to facilitate the writing of the boundary conditions and to transform the curvilinear field into a rectangular one.

Keywords: convective heat transfer; eccentric annuli; bicylindrical coordinates; vorticity-stream function formulation

1. introduction

The flow and heat transfer in enclosed space have been studied extensively in recent years becauseof their importance in engineering.

Nobari et al [1] studied numerically the fluid flow and heat transfer in curved eccentric annuli. A second order finite difference method based on the Projection algorithm is implemented to solve the governing equations including the full Navier–Stokes, the continuity, and the energy equations in a toroidal coordinate system. It is also shown that in contrast to straight eccentric annuli, heat transfer rates can be augmented in the eccentric curved annuli comparing with the straight concentric annuli at the large dean numbers. Shklyar et al [2] analyzed the convergence rate of a methodology for solving incompressible flow in general curvilinear co-ordinates. Overset grids (double-staggered grids type) type), each defined by the same boundaries as the physical domain are used for discretization. Kuehn and Goldstein [3] studied the effects of vertical eccentricity on heat transfer and obtained Mach-Zehnder interferograms for the thermal field in the annulus between vertically eccentric cylinders.

Nomenclature

a	defined constant in the eccentric coordinates (m)
C1	radius ratio
C2	eccentricity of the annular space formed by eccentric cylinders
c_p	Specific heat at constant pressure($j.kg^{-1}.k^{-1}$)
g	Gravitational acceleration ($m.s^{-2}$)
Gr	Modified Grashof number defined by: $Gr_m = \frac{g\, \beta\, D_h^4}{\lambda\, \upsilon^2}\, q$
h	Dimensional metric coefficient (m)
H	Dimensionless metric coefficient
Nu	Local Nusselt number
$\overline{Nu}$	Average Nusselt number
Pr	Prandtl number defined by $\Pr = \frac{\upsilon \rho c_p}{\lambda}$
q	Heat flux density ($W.m^{-2}$)
r_1, r_2	Inner and outer radius respectively (m)
S_Φ	Source term
T	Fluid's temperature (K)
T_1	Hot wall temperature (K)
T_2	Cold wall temperature (K)
ΔT	Temperature deference $\Delta T = T_1 - T_2$ (K)
V_η,V_θ	Velocity components η et θ ($m.s^{-1}$)
V	Velocity vector ($m.s^{-1}$)
Greek letters	
α	Angle of inclination (°)
β	Thermal expansion coefficient (K-1)
Γϕ	Diffusion Coefficient
λ	Thermal conductivity (W.m-1.K-1)
υ	Kinematic viscosity (m2. s-1)
ρ	Fluid density (kg. m-3)
η ,θ,z	bicylindrical coordinates
Ψ	Stream function (m2. s-1)

ω	Vorticity (s-1)
ϕ	General function
Π	Stress tensor

Chakrabarti et al. [4] obtained similar experimental results for the vertical eccentric configuration of cylinders. However, their study was for an annulus with a hot outer cylinder. In numerical work, most of the existing works on eccentric cylindrical annuli were also mainly done in terms of vorticity-stream function formulations [5, 6]. This requires an additional integral condition for the pressure to be single valued in a solution domain with unconnected enclosed boundaries [7]. Shu and Yeo [8] applied the global method of polynomial based differential quadrature (PDQ) and Fourier expansion-based differential quadrature (FDQ) to simulate the natural convection in an annulus between two arbitrarily eccentric cylinders. Their approach combined the high efficiency and accuracy of the differential quadrature (DQ) method with simple implementation of pressure single value condition.

2. Problem formulation and basic equations

Let's consider an annular space, filled with an incompressible Newtonian fluid, situated between two eccentric cylinders. Figure 1 represents a cross-section of the system.

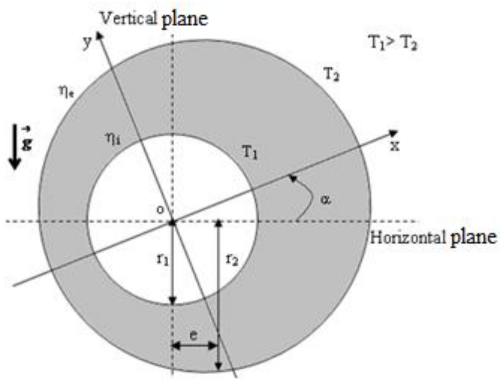

Fig. 1 a cross-section of the system

we impose on the inner cylinder a constant density and outer cylinder is isotherm which is held at temperature T_2. The physical properties of the fluid are constant, except the density ρ whose variations are at the origin of the natural convection. Viscous dissipation is neglected, just as the radiation (emissive properties of the two walls being neglected). We admit that the problem is bidimensionnal, permanent and laminar.

- *Continuity equation:*

$$div\vec{V} = 0 \tag{1}$$

- Momentum equation:

$$\frac{\partial \vec{V}}{\partial t}+(\vec{V}.\overrightarrow{grad})\vec{V}=\frac{\rho}{\rho_0}\vec{g}+\frac{\nabla P}{\rho_0} \qquad (2)$$

- Heat equation

$$\frac{\partial T}{\partial t}+(\vec{V}.\overrightarrow{grad})T=\frac{\lambda}{\rho\, c_p}\nabla^2 T \qquad (3)$$

The coordinates are:

$$\left.\begin{aligned} h&=\frac{a}{(ch(\eta)-cos(\theta))}\\ F(\eta,\theta)&=\frac{1-cos(\theta)ch(\eta)}{(ch(\eta)-cos(\theta))}\\ G(\eta,\theta)&=\frac{sin(\theta)sh(\eta)}{(ch(\eta)-cos(\theta))}\end{aligned}\right\} \qquad (4)$$

The equations (1), (2) and (3) becomes :

$$\frac{\partial}{\partial\eta}\left(hV_\eta\right)+\frac{\partial}{\partial\theta}\left(hV_\theta\right)=0 \qquad (5)$$

$$\frac{V_\eta}{h}\frac{\partial\omega}{\partial\eta}+\frac{V_\theta}{h}\frac{\partial\omega}{\partial\theta}=\frac{g\beta}{h}\left\{\left[F(\eta,\theta)cos(\alpha)+G(\eta,\theta)sin(\alpha)\right]\frac{\partial T}{\partial\eta}\right.$$
$$\left.+\left[F(\eta,\theta)sin(\alpha)-G(\eta,\theta)cos(\alpha)\right]\frac{\partial T}{\partial\theta}\right\}+\frac{\nu}{h^2}\left(\frac{\partial^2\omega}{\partial\eta^2}+\frac{\partial^2\omega}{\partial\theta^2}\right)$$

$$(6)\, V_\eta\frac{\partial T}{\partial\eta}+V_\theta\frac{\partial T}{\partial\theta}=\frac{\lambda}{\rho\, c_p}\frac{1}{h}\left(\frac{\partial^2 T}{\partial\eta^2}+\frac{\partial^2 T}{\partial\theta^2}\right)$$

(7)

The introduction of vorticity defined by:

$$\omega=-\frac{1}{h^2}\left(\frac{\partial^2\psi}{\partial\eta^2}+\frac{\partial^2\psi}{\partial\theta^2}\right) \qquad (8)$$

$$\left.\begin{aligned} h&=\frac{a}{(ch(\eta)-cos(\theta))}\\ F(\eta,\theta)&=\frac{1-ch(\eta)cos(\theta)}{(ch(\eta)-cos(\theta))}\\ G(\eta,\theta)&=\frac{sh(\eta)sin(\theta)}{(ch(\eta)-cos(\theta))}\end{aligned}\right\} \qquad (9)$$

We pass directly to the writing dimensionless equations, by posing the following dimensionless quantities:

$D_h = a$, $H=\frac{h}{D_h}$, $\quad V_\eta^+=V_\eta\frac{D_h}{\upsilon}$, $V_\theta^+=V_\theta\frac{D_h}{\upsilon}$, $\omega^+=\omega\frac{D_h^2}{\upsilon}$,

$$\psi^{+} = \frac{\psi}{\upsilon} \quad \text{et} \quad T^{+} = \frac{\lambda}{q\,Dh}(T - T_2)$$

The equations (5), (6), (7) and (8) becomes :

$$\frac{\partial}{\partial\eta}(H V_{\eta}^{+}) + \frac{\partial}{\partial\theta}(H V_{\theta}^{+}) = 0 \tag{10}$$

$$\frac{V_{\eta}^{+}}{H}\frac{\partial\omega^{+}}{\partial\eta} + \frac{V_{\theta}^{+}}{H}\frac{\partial\omega^{+}}{\partial\theta} = \frac{Gr_m}{H}\left\{ \left[F(\eta,\theta)\cos(\alpha) + G(\eta,\theta)\sin(\alpha)\right]\frac{\partial T^{+}}{\partial\eta} + \left[F(\eta,\theta)\sin(\alpha) - G(\eta,\theta)\cos(\alpha)\right]\frac{\partial T^{+}}{\partial\theta} \right\} + \frac{1}{H^2}\left(\frac{\partial^2\omega^{+}}{\partial\eta^2} + \frac{\partial^2\omega^{+}}{\partial\theta^2} \right) \tag{11}$$

$$HV_{\eta}^{+}\frac{\partial T^{+}}{\partial\eta} + H V_{\theta}^{+}\frac{\partial T^{+}}{\partial\theta} = \frac{1}{P_r}\left(\frac{\partial^2 T^{+}}{\partial\eta^2} + \frac{\partial^2 T^{+}}{\partial\theta^2}\right) \tag{12}$$

$$\omega^{+} = -\frac{1}{H^2}\left[\frac{\partial^2\psi^{+}}{\partial\eta^2} + \frac{\partial^2\psi^{+}}{\partial\theta^2}\right] \tag{13}$$

The boundary conditions are the following ones:

- inner cylinder wall Condition ($\eta=\eta i$=constant):

$$V_{\eta}^{+} = V_{\theta}^{+} = \frac{\partial\psi^{+}}{\partial\theta} = \frac{\partial\psi^{+}}{\partial\eta} = 0 \quad ,$$

$$\omega^{+} = -\frac{1}{H^2}\left[\frac{\partial^2\psi^{+}}{\partial\eta^2} + \frac{\partial^2\psi^{+}}{\partial\theta^2}\right]$$

$$q_{constant} = -\lambda\frac{1}{h}\left.\frac{\partial T}{\partial\eta}\right|_{\eta=\eta_i}$$

$$\frac{1}{H}\left.\frac{\partial T^{+}}{\partial\eta}\right|_{\eta=\eta_i} = -1$$

- Outer cylinder wall Condition ($\eta=\eta_e$=constant):

$$V_{\eta}^{+} = V_{\theta}^{+} = \frac{\partial\psi^{+}}{\partial\theta} = \frac{\partial\psi^{+}}{\partial\eta} = 0$$

$$\omega^{+} = -\frac{1}{H^2}\left[\frac{\partial^2\psi^{+}}{\partial\eta^2} + \frac{\partial^2\psi^{+}}{\partial\theta^2}\right]$$

$$T_2^{+} = 0$$

The temperatures distribution obtained local Nusselt number value relation:

$$Nu = -\frac{1}{h^{+}T^{+}(\eta_i,\theta)}\frac{\partial T^{+}}{\partial \eta}\bigg|_{\eta=cste} \tag{14}$$

The average Nusselt number is:

$$\overline{Nu} = \frac{1}{\theta_{NN}-\theta_1}\int_{\theta_1}^{\theta_{NN}} Nu d\theta \tag{15}$$

3. *Numerical Formulation*

To solve the equations (11) and (12) with the associated boundary conditions, we consider a numerical solution by the method of finite volumes, exposed by Patankar [9]. For the equation (13), we consider a numerical solution by the method of the centered differences, exposed by Nogotov [10].

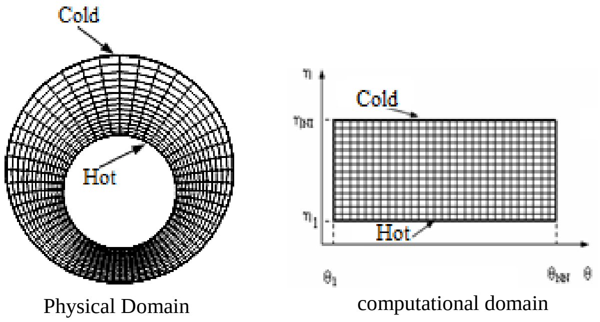

Fig. 2 physical and computational domain

4. *Discretization equation transfer of a variable φ*

The general differential equation is**:**

$$\frac{\partial}{\partial \eta}\left(H V_{\eta}^{+}\varphi - \Gamma_{\varphi}\frac{\partial \varphi}{\partial \eta}\right) + \frac{\partial}{\partial \theta}\left(H V_{\theta}^{+}\varphi - \Gamma_{\varphi}\frac{\partial \varphi}{\partial \theta}\right) = S_{\varphi} \tag{16}$$

We illustrate sources and diffusion coefficients in table 1

Table 1
Sources and diffusion coefficients

φ	Γ_φ	S_φ
T^+	$1/Pr$	0
ω^+	1	$HGr_m\{[F(\eta,\theta)cos\alpha + G(\eta,\theta)sin\alpha]\frac{\partial T^+}{\partial \eta} + [F(\eta,\theta)sin\alpha - G(\eta,\theta)cos\alpha]\frac{\partial T^+}{\partial \theta}\}$

The discretization equation is obtained by integrating the conservation equation over the control volume shown in figure 3, after some manipulations we have the final discretization equation:

$$a_P\,\varphi_P = a_N\,\varphi_N + a_S\,\varphi_S + a_E\,\varphi_E + a_W\,\varphi_W + b \tag{17}$$

The equation coefficients are well defined in Patankar [6]. The power law scheme is used to discretize the convective terms in the governing equations.

5. Results and discussion

We consider three annular spaces formed by eccentric cylinders with three values of inclination angle (α=0°, 45° and 90°) and relative eccentricity (C_2=0.4).

6. Grid study

In this study several grids were used arbitrarily, to see their effect on the results. Table 2 shows us the variation of average Nusselt number and the maximum of the stream function value according to the number of nodes for each grid. We choose the grid (101x111).

Table2
Variation of average Nusselt number and the maximum of the stream function value according to the number of nodes

	$Gr=10^4$		$Gr=5.10^4$		$Gr=10^5$		$Gr=10^6$	
	ψ_{max}	|Er| %	ψ_{max}	|Er|	ψ_{max}	|Er|	ψ_{max}	|Er|
21x31	1.32	-	5.23	-	8.86	-	34.99	-
31x41	1.34	1.49	5.28	0.94	9.04	1.99	34.41	1.65
41x41	1.34	0.00	5.29	0.19	8.93	1.21	33.99	1.65
51x61	1.33	0.75	5.26	0.57	8.81	1.34	33.58	1.17
61x71	1.31	1.50	5.20	1.14	8.72	1.02	33.29	0.83
71x81	1.29	1.52	5.15	0.96	8.64	0.91	33.06	0.67
81x91	1.28	0.77	5.09	1.16	8.56	0.92	32.85	0.63
91x101	1.27	0.78	5.04	0.98	8.46	1.16	32.64	0.63
101x111	1.25	1.57	4.98	1.19	8.37	1.06	32.45	0.58
111x121	1.24	0.80	4.93	1.00	8.30	0.83	32.29	0.49

7. Numerical code validation

Kuehn et al. (1976) developed a numerical study on natural convection in the annulus between two concentric cylinders and horizontal with a radius ratio was taken equal to 2.6, they calculated a local equivalent thermal conductivity, defined as the report of a temperature gradient in a convective heat exchange on a temperature gradient in an exchange conduction:

$$\lambda_{eq} = \frac{\left.\frac{\partial T^+}{\partial \eta}\right|_{convection}}{\left.\frac{\partial T^+}{\partial \eta}\right|_{conduction}}$$

They calculated an average value of the conductivity.

We applied our computer code to this case and we compared the average value of our results with theirs, we notice that they are in concord. Table 3 illustrates this comparison well.

Table3

Comparaison of the average thermical conductivity of Kuehn with our results

Numerical study					
	Pr	0.7	0.7	0.7	0.7
	Ra	10^2	10^3	6×10^3	10^4
Inner wall	Kuehn	1.00	1.08	1.73	2.01
	our calculs	1.00	1.06	1.73	2.06
	|E (%)|	0	1.4	0.3	2.8
Outer wall	Kuehn	1.00	1.08	1.73	2.01
	our calculs	1.00	1.06	1.73	2.07
	|E (%)|	0	1.7	0.05	3.5

8. *The thermal condition: isotherm inner cylinder*

- ***Influence of the inclination angle α***
- ***Isotherms and stream lines***

Figures 4 and 5 represent the isotherms and the streamlines for different values of the inclination angle when the relative eccentricity C_2=0.4 and $Gr_m=10^7$.

These figures show that the structure of the flow is bicellular. The flow turns in the trigonometrically direction in the left side and in the opposite direction in the right one.

When the inclination angle is equal to 0°, the isothermal lines take the form of a mushroom. The temperature distribution is decreasing in the hot wall towards the cold wall. The direction of the deformation of the isotherms is consistent with the direction of rotation of the streamlines. In laminar flow, we can say that under the action of the movement of particles flying from the hot wall at the symmetry axis, the isothermal lines are away from the wall there. The values of the Stream function increase which means that convection increases.

For α=45°, the region of the constriction of the annular space moves down wardly against by the expansion region moves upward. Figure 4 show that for large values of modified Grashof number, the fluid motion is more important in the area of enlargement.

For α=90°, the isotherms of figure 5 are almost parallel to the walls. Nevertheless there is a movement of the fluid: the particles, which warm up on the hot wall, tend to rise along this one, then to go down again along the cold wall. Thus the flow is organized in two principal cells which turn very slowly in opposite directions.

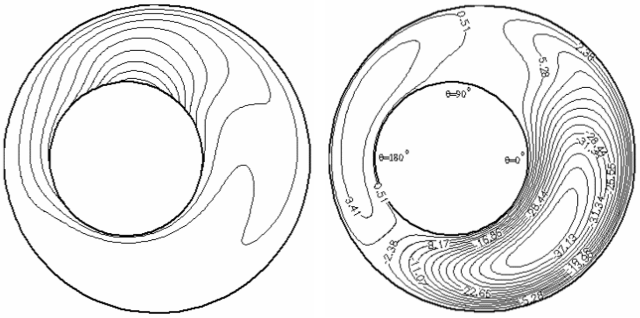

Fig. 3 Isotherms and streamlines for C2=0.4, α=0° and $Gr_m=10^7$

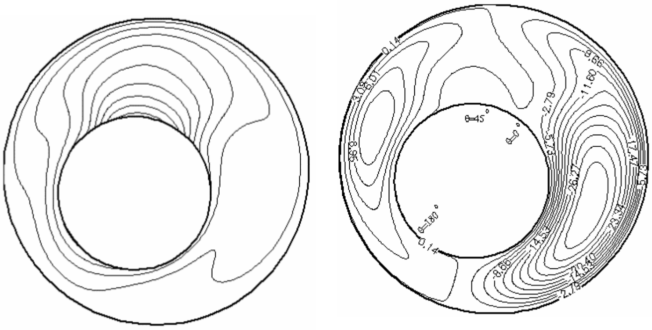

Fig. 4 Isotherms and streamlines for C2=0.4, α=45° and $Gr_m=10^7$

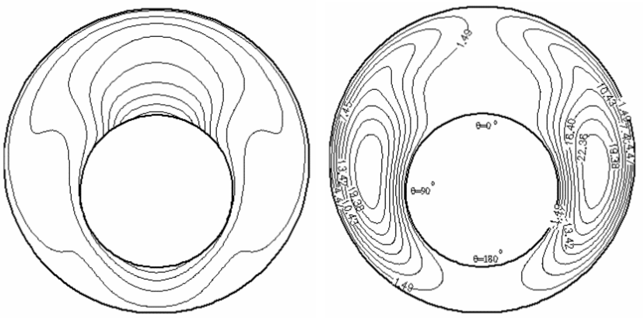

Fig. 5 Isotherms and streamlines for C2=0.4, α=90° and $Gr_m=10^7$

9. Conclusion

We have established a mathematical which based on the assumption of Boussinesq and the two-dimensionality of the flow. We have developed a program of numerical calculation, based on the finite volume method, which determines the thermal and dynamic fields in the fluid and the dimensionless numbers of local and average Nusselt numbers on the active walls of the enclosure , depending on the quantities characterizing the state of the system. The influence of the inclination of the system, on the flow was particularly examined.

The results of numerical simulations have shown that the transfers are better when our system has elements of symmetry.

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