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ANALYSIS AND NUMERICAL MODELLING OF CERAMIC PIEZOELECTRIC BEAM BEHAVIOR UNDER THE EFFECT OF EXTERNAL SOLICITATIONS

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ABSTRACT

The piezoelectric materials have become indispensable in many technological applications. These materials have an inverse piezoelectric effect which allows them to control the form and to present any neither noise nor vibration at any time or position on the structure. In this study we are interested in the bending behavior analysis and modelling of a ceramic beam under external solicitations using numerical simulations based on the finite element methods. The modelling permit to simulate the deformations in a piezoelectric ceramic beam subjected to an electric field and to simple mechanical stress taking into account the electromechanical coupling. It has been found that the obtained analytical results are in a very good agreement with those obtained by numerical modeling. As a result, the interest of such modelling analysis allows the design, the conception and the optimization of mechanical systems based on piezoelectric elements. These materials known as smart or "intelligent" materials, are often used to measure and/or to control finite deformations or vibrations in mechanical systems, so that to prevent their plastic deformations or their total failure.

Keywords: *Piezoelectric material, ceramic beam, intelligent structure, finite elements modelling.*

NOMENCLATURE

Symboles :

E_i the electric field

D_i the induced electric displacement.

L the length.

S the section (S = b.h)

E the Young's modulus

I the bending moment of inertia

G the Shear's modulus

d_{ji} the Piezoelectric's coefficient

y_e the deflection caused by electric field applied on a piezoelectric beam

y_F the deflection caused by a concentrated force on a piezoelectric beam

Lettres grecques :

ρ the density

ε_{ij} the strain tensors

σ_{ij} the stress tensors

δ_T the global deformation

ν the Poisson's ratio

1. INTRODUCTION

The piezoelectric ceramics such as titanozirconate lead (PZT) [1]-[3] are used in many industrial applications such as sensors, actuators and more generally in the so-called "smart" structures [4]. Indeed, the piezoelectric elements having the shape of mini bar, beam, plate or others are used perfectly like sensors of displacement and generators of force or energy to control the phenomena of small deformation, vibration or even of vibro- acoustic. The deformation direction in these elements is mainly dominated by the sizes of the piezo- electromechanical coupling d_{31}, d_{32} and d_{33}. This deformation mode depends on the way of application of the electric field. In the case of a piezoelectric beam, the dominant deformation is perpendicular to this applied field as shown on Figure 2. In this work, the piezoelectric beam is made of two uni-axial piezoelectric material layers (PZT) of opposite polarity. These types of beams generally called bimorph beams can be used for the micro positioning and for the precise pointing of components. When an electric force is applied transversely to the thickness, the internal deformations of the two piezoelectric layers induce control forces that beam the intelligent structure. In the piezoelectric beams, the static linear behaviour subjected to mechanical, electrical and thermal stresses has been studied since a long time [5]. However, the physical modelling really began with Smits and Choi work [6]. In the case of an electrical excitation, Rogacheva and Al [7], then Chang and Chou [8] have showed that the piezoelectric constant d, depend on the alimentary frequency. Finally, Weinberg [9] extended the physical formulation from piezoelectric beams to the layered ones. The piezoelectric beams can be modeled in the same way as cantilever beams. The electric force can be assimilated by a bending force applied in the plane of the middle line. In a first approximation, the effect of the actuation force can be represented by a single force applied on a point of the cantilever. Our main objective here is to exploit the properties of piezoelectric materials in their form of bimorph beam, in the context of the design and the development of a micro manipulation tool (Figure 1). The modelling has been performed with the Ansys® software, in order to obtain a various magnitude of static deflections of different PZT beams.

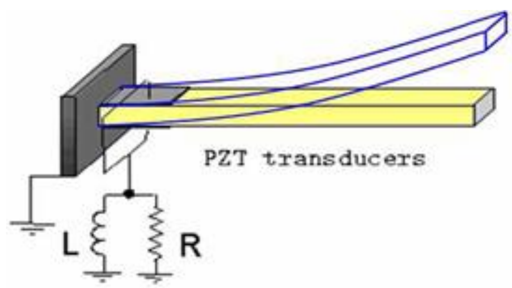


FIGURE 2. Operating principle of a piezoelectric beam of a cantilever type

2. MODELLING OF A PIEZOLECTRIC BIMORPH BEAM

The piezoelectric ceramics subjected to a voltage generate very low displacements. These elements are often presented in the shape of cantilever beams, mainly fixed at one end, while the other end is free from any vertical movement (Figure 2). The cantilever will be suspended on the top of an electrode of actuation fixed on the substrate in order to deform the beam. In the theory of linear piezoelectricity, the equations of linear elasticity are coupled to the electrostatic equations by means of piezoelectric coefficients [10]-[11], that is:

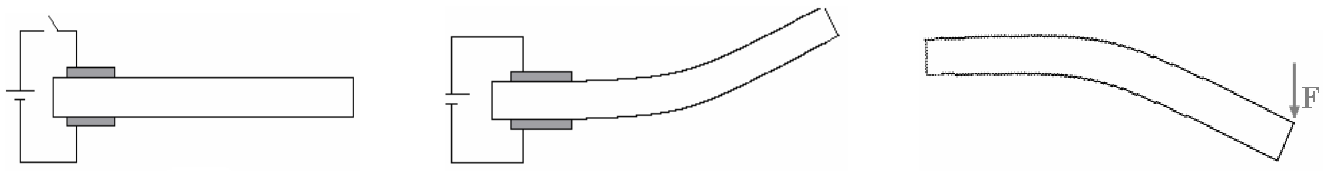
$$\varepsilon = s^{E}.\sigma + d.E \tag{1}$$

$$D = d.\sigma + \varepsilon^{\sigma}.E \tag{2}$$

With ε_{ij} and σ_{ij} are the strain and stress tensors; E_i is the electric field and D_i is the induced electric displacement.

2.1 ANALYTICAL MODELLING

Consider a piezoelectric bimorph beam fixed at one end and submitted independently either to an electric field or to a force at the other end (Figure 3). The physical characteristics of this beam are the following: L the length, ρ the density, S the section (S = bh), E its Young's modulus and I its bending moment of inertia.



(a) Specimen of PZT. Voltage OFF (b) Specimen of PZT. Voltage ON (c) Specimen of PZT. Stress imposed

FIGURE 3. Deformation of a bimorph beam under various types of solicitations

From equation (1), the most interesting effective coefficients which are of big interest to us appear in the following equation:

$$\varepsilon_1 = s_{13}^{E}\sigma_1 + d_{31}E_3 \qquad (3)$$

With: $\varepsilon_1 = \dfrac{\partial^2 y}{\partial x^2}$

Initially, if only the electric field is applied (Figure 3b), the beam will be deformed and we assume that there is a free volume stress then equation (3) reduces to:

$$\varepsilon_1 = d_{31}E_3 \qquad (4)$$

As a result:

$$y_e(x) = \frac{1}{3}d_{31}\frac{Vx^2}{h^2} \qquad (5)$$

In a second step, if only a concentrated force is applied to the free edge of the beam (Figure 3c), the solution is solved analytically using the strength of materials. Thus, from the integration of the equation of bending moments, it is possible to establish the equation of the deflection caused by a concentrated force on a piezoelectric beam. The deflection caused on all points of the cantilever will be by:

$$y_F(x) = -\frac{F}{6EI}x^2(x-3L) \qquad (6)$$

Where E is the Young's modulus of the material constituting the beam and I is the quadratic moment of inertia of the cantilever. Knowing that, the global deformation (δ_T) of the piezoelectric beams in bending is the summation of the two deflections. This answer is then this generated mechanically (y_F) on one hand and electrically (y_e) on the other. We finally get the total deformation at x = L:

$$\delta_T = -\frac{Fl^3}{3EI} + d_{31}\frac{Vl^2}{h^2} \qquad (7)$$

2.1 FINITE ELEMENTS MODELLING

The pure bending analysis of a bimorph beam has been realised by numerical simulations using the finite element methods to compare these numerical results to those obtained analytically. The piezoelectric beam is modelled only by the elements which take into account the piezoelectric effects. It has been discretized by twenty elements in the direction length i.e. of two elements per linked axial layer of the PZT material. The electromechanical characteristics corresponding to the behaviour of PZT material gathered in table 1. The geometrical specimen and its boundary condition are represented on Figure 4.

Material	PZT-189p
Young's modulus (E)	2.109 N/m^2
Shear’s modulus (G)	0,75.109 N/m^2

Poisson's ratio (ν)	0.29
Piezoelectric's coefficient (d_{31})	2,5 .10-11 C/N
Piezoelectric's coefficient (d_{32})	0,3 .10-11 C/N
Piezoelectric's coefficient (d_{33})	3,6 .10-11 C/N

TABLEAU 1. Mechanical and piezoelectric properties of the pzt material

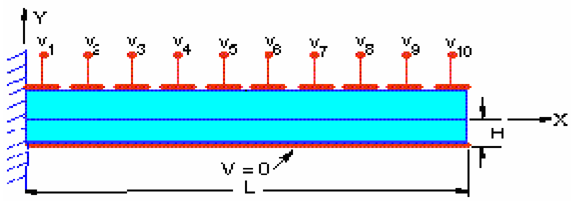
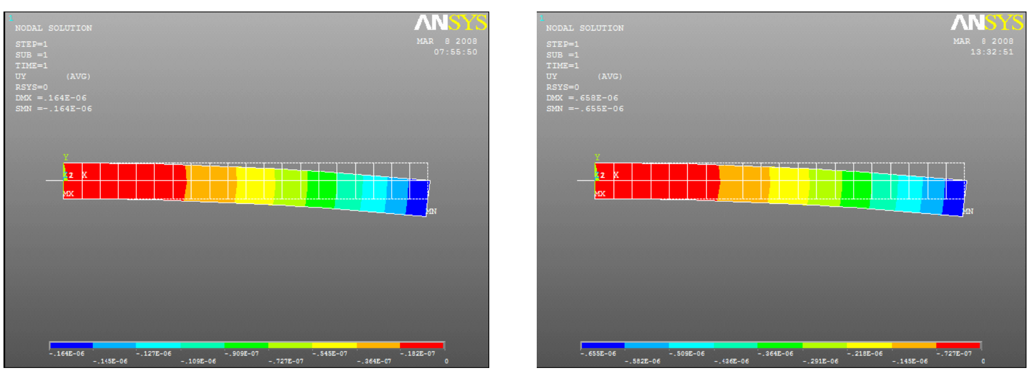


FIGURE. 4. Geometry of the embedded-free piezoelectric beam (L = 10mm, H = 0.25mm).

The applied tension on the cantilever structure is modelled by ten electrodes (Figure 4). The value of this tension depends on the rigidity of the beam, i.e. more than the stiffness constant is higher more than the actuation load is significant, in order to fight against the recall force. The beam is then subjected by imposing electrical tensions on the electrodes and the structural deformation is characterized by observing the displacement of the point at the end of the beam.

3. RESULTS AND DISCUSSION OF A PIEZOLECTRIC BIMORPH BEAM

The bending analysis of a bimorph beam has been considered using numerical simulations [12] based on the finite element methods and compared with those of obtained analytically. The results obtained by the finite element method allow to evaluate the displacements U_y of all the points of the deformed structure and they are represented on Figure 5 in term of iso value of displacements for two values of electric fields applied separately (50 and 200 volts). It is clear to note that these results are in conformity with those concerning the geometrical evolutions of the specimens during deflection.



(a) (b)

FIGURE 5. Deformation of bimorph beam actuated by electrical extensions: (a) 50 volts (b) 200 volts

As illustrated in Figure 6, the simulation performed with the piezoelectric material (PZT) indicates a good agreement with the analytical solution in terms of changes of the deflection at any position of the beam. The numerical curve is almost equivalent to the analytical one. Therefore, the simulation results are correct.

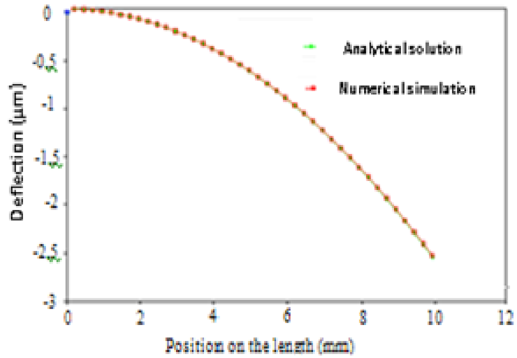


FIGURE 6. Deflection calculation of an imposed cantilever beam theory and numerical simulation

Moreover, it is possible to draw on the same graph all the deflection curves of the bimorph beam obtained for different values of electric field. The superposition of these curves permits to quantify the value of deflection at the end of the cantilever. It has been found that the deflection is directly proportional to the value of the applied electric field (Figure. 7).

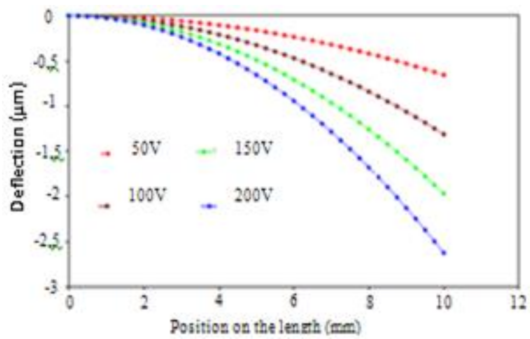


FIGURE 7. Deflection of the cantilever beam induced by an electric field

In addition, the deflection of this beam can be obtained in term of different types of loading electric and mechanical applied independently, in order to compare what A makes it possible to compare the final pace of the deformed structure. Initially, an electric field is imposed and in the second time, a constant force is applied at the end of the beam, and the results are represented on figure 8. On this figure, we can observe a variation of the form, this difference can be attributed to the positioning and the numbers of electrodes stuck on the surfaces of the bimorph beam. Moreover, we observe that two displacements at the end of the beam are similar.

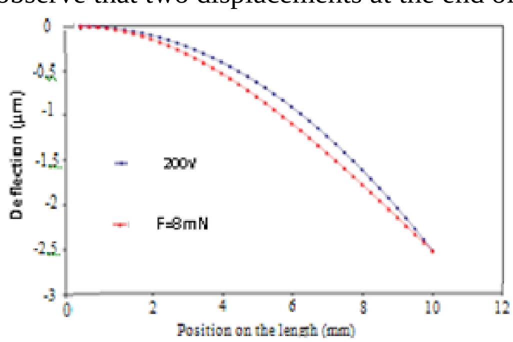


FIGURE 8. Comparison of the deformation evolution of a bimorph beam under various electrical and mechanical solicitations.

4. CONCLUSION

A finite element modeling of piezoelectric beams in small displacement has been developed and presented in this work. The electromechanical coupling has been considered. Taking into account the electric potentials as additional degrees of freedom, it was possible to simulate the static or the dynamic behavior of a piezoelectric beam under the effect of external applied stresses. We have shown that the numerical simulations reproduce well the displacement at the free end of the PZT beam under the influence of electric fields or imposed forces. Moreover, the obtained results allow us to know the magnitude of the imposed force which is equivalent to the applied electrical field for a given displacement. This analysis is necessary for the design, the optimization and the development of systems for the micromanipulation and the assembly of micro objects used in nano and micro biological fields. It is also applicable to the structures of any shape and that may include piezoelectric components in order to control their shape, their displacement and also their slow vibration. As a result this can give the opportunity to develop the micro process based on the analysis of these types of bimorph beam subjected to electric fields in different senses.

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