

# Modeling of Heat Transfer by Laminar Natural Convection of a nanofluid in a Solar Water Heater Enclosure

Mabrouk GUESTAL, Mahfoud KADJA

Laboratory of Applied Energetics and Pollution, Department of Mechanical Engineering  
University of Frères Mentouri Constantine, Constantine 25000, Algeria  
Corresponding author: gamabrouk @yahoo.fr

**Abstract** - We numerically study the laminar natural convection of a water copper nanofluid in a solar water heater enclosure with heating through the solar collector wall being at constant temperature  $T_c$ . This solar collector is connected with a solar thermal storage tank having a rectangular form with the vertical wall left cold at constant temperature  $T_f$  while the unheated parts of the enclosure (solar water heater) were considered adiabatic. For the purpose of analyzing the effect of the use of a nanofluid on heat transfer by natural convection, the volume fraction of the particles is varied in the range of 0 to 0.25. The permanent forms of the Navier-Stokes equations in two dimensions and the conservation equations of mass and energy were solved by the finite volume method. The SIMPLE algorithm was used to allow pressure-velocity coupling. The Rayleigh number  $Ra$  was varied in the range  $10^3$ - $10^6$ .

**Keywords:** Laminar natural convection; solar water heater; nanofluid; finite volume.

## Nomenclature

### Symbol

$g$  acceleration of gravity,  $m\ s^{-2}$   
 $L1$  width of thermal storage tank,  $m$   
 $H$  the enclosure height,  $m$   
 $H1$  solar collector output section height,  $m$   
 $L$  enclosure width,  $m$   
 $k$  thermal conductivity of the fluid,  $Wm^{-1}K^{-1}$   
 $p$  pressure,  $Pa$   
 $P$  dimensionless pressure  
 $u$  horizontal velocity,  $m\ s^{-1}$   
 $v$  vertical velocity,  $m\ s^{-1}$   
 $U$  dimensionless horizontal velocity  
 $V$  dimensionless vertical velocity  
 $T_f$  temperature of cold surface,  $K$   
 $T_c$  temperature of hot surface,  $K$   
 $x, y$  coordinates,  $m$   
 $\bar{h}$  average heat transfer coefficient,  $Wm^{-2}K^{-1}$

$\bar{Nu}$  average Nusselt number .

$Ra$  Rayleigh number,  $g\ \beta_f\ H^3\ (T_c - T_f) / \nu_f\ \alpha_f$

$Pr$  Prandtl number,  $Pr = \nu_f / \alpha_f$

### Greek symbols

$\alpha$  thermal diffusivity,  $m^2\ s^{-1}$   
 $\nu$  kinematic viscosity,  $m^2\ s^{-1}$   
 $\rho$  density,  $kg\ m^{-3}$   
 $\phi_v$  volume fraction of the nanoparticles  
 $\theta$  dimensionless temperature  
 $\Omega$  inclination angle of the solar collector  
 $\beta$  thermal expansion coefficient at constant pressure,  $K^{-1}$

### Indices / Exponents

$f$  base fluid  
 $nf$  nanofluid  
 $s$  solid

## 1. Introduction

Today a lot of industrial applications are based on the heat transfer by natural convection phenomenon such as boilers, heat exchangers, cooling systems for electronic components, solar thermal water heaters..... This is why we find many numerical and experimental research focused on improving the heat transfer by natural convection in these energy systems. Recently, ideas for improvement of heat transfer involve mainly the physicochemical nature of convective fluid, since the thermal conductivity of the fluid is relatively low compared to that of the solid. These ideas are exploited by injecting inside the fluid a quantity of solid particles of nanometer sizes in order to

increase its effective thermal conductivity. The mixture thus obtained is called nanofluid. Several recent studies have addressed this problem. Wang et al. [2] measured the thermal conductivity of nanofluids containing  $\text{Al}_2\text{O}_3$  and  $\text{CuO}$  nanoparticles scattered in various base fluids. The study showed that the thermal conductivity of these nanofluids increases with the increase of the volume fraction of the nanoparticles in the base fluid, and for a given volume fraction, the noticed increase in the thermal conductivity is different depending on the various base fluids. Das et al. [3] studied the influence of temperature on the thermal conductivity for the nanofluids ( $\text{Al}_2\text{O}_3$ +water) and ( $\text{CuO}$ +water). The results show that the thermal conductivity of the nanofluids increases linearly with increasing temperature. Khanafer et al. [4] have numerically studied natural convection inside a rectangular enclosure filled with nanofluids in which one of the vertical walls is kept cold, and the other wall is kept hot, while the horizontal walls are insulated. The results show that for the range of the Grashof number  $10^3 \leq \text{Gr} \leq 10^6$ , the average Nusselt number increases with the increase of the volume fraction of the nanoparticles. Hakan et al. [5] studied the effect of using different nanofluids on the temperature distribution in a partially heated cavity. The results show that the heat transfer increases with the increase of the value of the Rayleigh number. Mansour et al. [6] have studied numerically heat transfer by natural convection in a cavity in the form of T filled with a nanofluid ( $\text{Cu} + \text{water}$ ). The results show that the average Nusselt number increases with the increase of the Rayleigh number and the volume fraction of copper nanoparticles.

In this work, we present a numerical study of the improvement of heat transfer by laminar natural convection with the use of water-copper nanofluids. The study configuration is a solar thermal water heater enclosure subjected to heating under constant temperature  $T_c$  at solar collector level. Unheated parts of the enclosure were considered adiabatic. The vertical wall was maintained cold at constant temperature  $T_f$ . The purpose of the study is to examine the effect of volume fractions [ $\phi_v=0$  (corresponding to pure fluid) to  $\phi_v =0,3$ ] for different Rayleigh numbers ( $10^3 \leq \text{Ra} \leq 10^6$ ) on fluid flow and heat transfer by laminar natural convection in the enclosure. The results are presented as isothermal curves, streamlines, and variation of the average Nusselt number as a function of Rayleigh number and volume fraction.

## 2. Mathematical formulation

### a. Physical model and governing equations

This study focuses on the laminar natural convection of a water-copper nanofluid in a solar thermal water heater enclosure, which is shown in “Figure 1”.

We assume that the enclosure is infinitely long in the z direction. The solar collector wall is maintained at a constant hot temperature  $T_c$ . The solar thermal storage tank left vertical wall is maintained at a low constant temperature  $T_f$ . The remaining unheated parts of the enclosure are considered adiabatic.

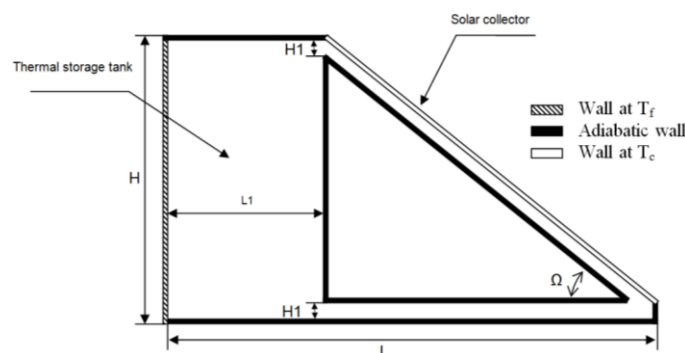


Figure 1 : The simulated enclosure

For purposes of this analysis, the volume fraction will be varied in the interval  $\varphi_v = 0-0.25$  [ $\varphi_v=0$  corresponds to pure fluid], The study is performed in the range of the Rayleigh number of  $10^3$  to  $10^6$ . For a simple formulation of the problem, we made some assumptions: that the fluid is Newtonian, that the flow is stationary and that the Boussinesq approximation applies. Indeed, we assume that the influence of density variation is taken into account only via the volume forces. The fluid density varies linearly with temperature and is given by the following formula:

$$\rho = \rho_0 \left[ 1 - \beta (T - T_0) \right] \quad (1)$$

The selected dimensionless parameters are:

The mass conservation equation, the momentum and energy equations, expressed in dimensionless variables are:

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = 0 \quad (2)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (3)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} Ra Pr \theta \quad (4)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (5)$$

The density and specific heat of the nanofluid are calculated according to [7] from:

$$(\rho C_p)_{nf} = (1 - \varphi_v)(\rho C_p)_f + \varphi_v(\rho C_p)_s \quad (7)$$

The thermal expansion coefficient of a nanofluid is obtained from the formula [7]:

$$(\rho\beta)_{nf} = (1 - \varphi_v)(\rho\beta)_f + \varphi_v(\rho\beta)_s \quad (8)$$

The thermal diffusivity of a nanofluid is given by:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (9)$$

In this study, the Brinkman model is used to find the dynamic viscosity of the nanofluid. It is obtained from the following equation [7]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi_v)^{2.5}} \quad (10)$$

The effective thermal conductivity of the nanofluid is determined using the Maxwell model [7]. For a suspension of nanoparticles of spherical shapes in a base fluid, the expression is:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f + 2\phi_v(k_s - k_f)}{k_s + 2k_f - \phi_v(k_s - k_f)} \quad (11)$$

### b. Heat transfer

The heat transfer is characterized by the average Nusselt number which is calculated for the heated wall using the formula:

$$\overline{Nu} = \frac{\bar{h}\sqrt{(H - H_1)^2 + (L - L_1)^2}}{k_f} \quad (12)$$

### c. Dimensionless boundary conditions

The different boundary conditions in dimensionless form are shown in Figure 2.

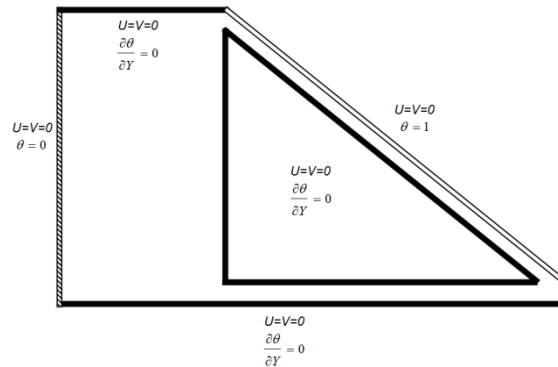


Figure 2 : Boundary conditions in dimensionless form.

### 3. Numerical procedure

The mass conservation equation along with the momentum and energy equations are solved numerically using the commercial software Fluent which is based on the finite volume method [1]. The influence of the number of nodes on the accuracy of the results for example in the case of  $Ra = 10^3$  and  $\phi_v = 0$ , is illustrated in Figure 3 which shows heat transfer through the active wall "that is to say the heated wall" of the enclosure.

From the grid of 22096 nodes onwards the average Nusselt number becomes constant. Therefore, all the simulations reported hereafter used an unstructured mesh consisting of 22096 nodes and triangular cell type (see fig.4). All the calculations in this grid independence study were made for the case of  $Ra = 10^3$  and  $\phi_v = 0$ .

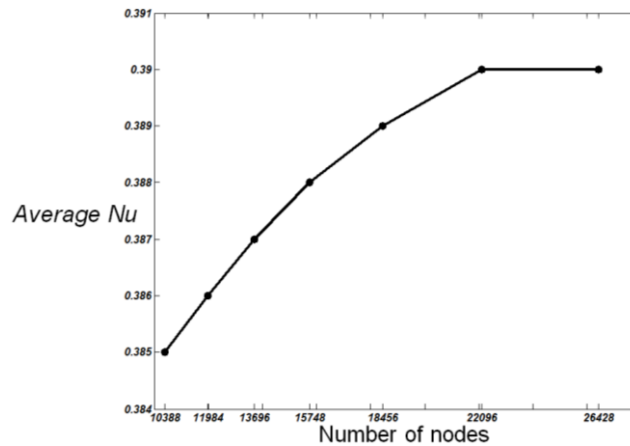


Figure 3 : Average Nusselt number along the heated wall for  $\phi_v = 0$ , and  $Ra = 10^3$

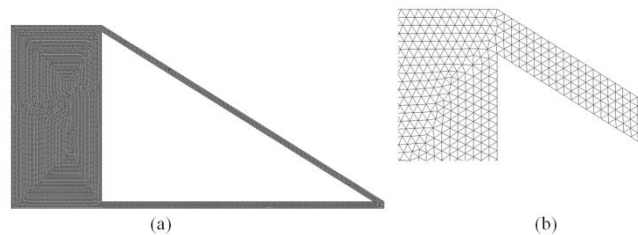


Figure 4 : The final mesh (a) and enlargement (b)

The calculation algorithm uses a pressure based method which solves the equations of the mathematical model sequentially. The SIMPLE algorithm is used for pressure-velocity coupling. The discretization of the convective terms in the conservation equations is made with the "QUICK" scheme, while the centered scheme is used to discretize the diffusive terms. The interpolation of the pressure is linear and uses the distances between nodes and those between nodes and the faces of the control volumes.

The convergence for all equations is reached when the sum of the normalized residuals at each node of the computational domain and for all algebraic equations- obtained after discretization- becomes less than  $10^{-3}$ . This usually requires a number of iterations 296 on a HP type laptop .

#### 4. Results and discussion

In this study, we investigated the effects of Rayleigh number ( $10^3 \leq Ra \leq 10^6$ ) and the volume fraction ( $0 \leq \phi_v \leq 0.25$ ) on fluid flow and heat transfer in the enclosure.

##### a. Thermal fields

This field is shown by the temperature contours in Figure 5 for a Rayleigh number which varies in the range  $10^3$ - $10^6$ , and for a volume fraction  $\phi_v$  which varies from 0 to 0.25.

The recovered heat through the hot wall is transported by natural convection to the upper portion of the solar thermal storage tank via the molecules of fluid which were beforehand in the solar collector. The heat is then discharged through the cold wall of the tank. The remaining cold fluid is then transported to the bottom of the solar collector and the heating cycle is renewed indefinitely at other times.

**- For Ra fixed and  $\phi_v$  varied from 0 to 0.25:**

It can be noticed that for each value of Ra the temperature contours are almost identical.

**- For  $\phi_v$  fixed and Ra varied from  $10^3$  to  $10^6$ :**

By comparing the isotherms shown in Figure 5, for different values of Ra, it may be noticed that when the Ra increases, the horizontal temperature gradient becomes vertical. The isotherms approach each other in the area near the exit of the solar collector which means that the temperature gradients become higher near the hot fluid inlet in the solar thermal storage tank. At the solar collector input we note that the thermal boundary layers become thinner and the isotherms become stratified for  $Ra = 10^6$ .

**b. Dynamic fields**

This field is represented by the contours of the streamlines in Figure 6, for a Rayleigh number which ranges between  $10^3$  and  $10^6$ , and for a volume fraction  $\phi_v$  which varies from 0 to 0.25. Generally speaking we note a recirculating fluid forming a cell in the thermal storage tank, which rotates in the anticlockwise direction.

**- For Ra fixed and  $\phi_v$  varied from 0 to 0.25:**

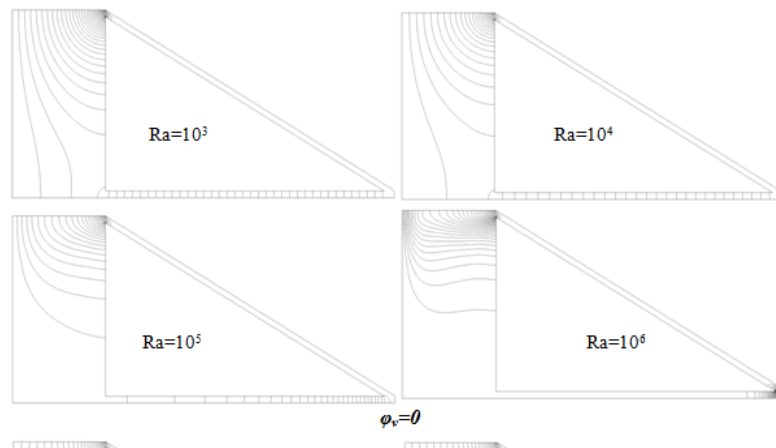
It can be noticed that for each value of the Ra number the streamlines are almost identical.

**- For  $\phi_v$  fixed and Ra varied from  $10^3$  to  $10^6$ :**

For a given value of  $\phi_v$  it can be noticed that with the increase of Rayleigh number, the intensity of recirculation inside the enclosure increases and the center of the rotating fluid cell moves towards the upper portion of the cold wall. For  $Ra = 10^6$  it can be noticed that the diameter of the rotating cell decreases.

**c. Nusselt number**

The evolution of the average Nusselt number as a function of the Rayleigh number for different values of  $\phi_v$  is shown in Figure 7. For  $Ra = [10^3 \text{ to } 10^4]$  we note that the average Nusselt number remains constant for each value of  $\phi_v$ . Generally speaking for  $Ra = [10^4 \text{ to } 10^6]$  the average Nusselt number increases with increasing  $\phi_v$  and for a given value of  $\phi_v$ , the average Nusselt number increases when Ra increases.



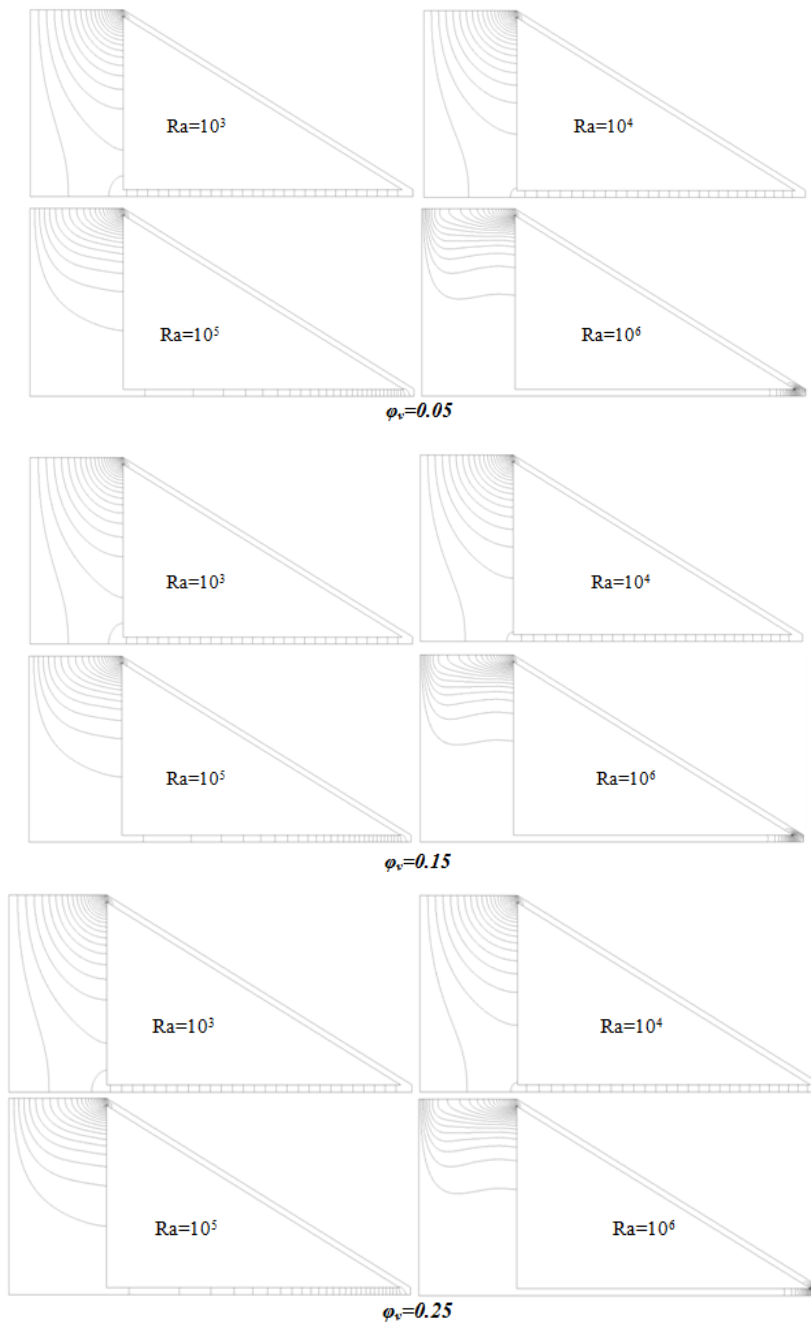
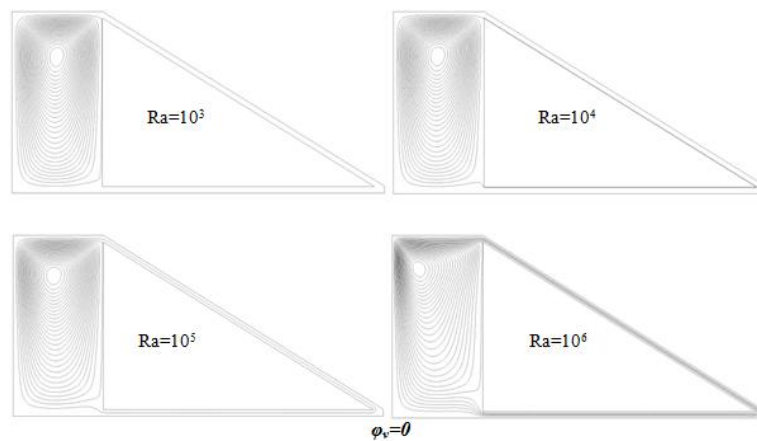


Figure 5 : Temperature contours



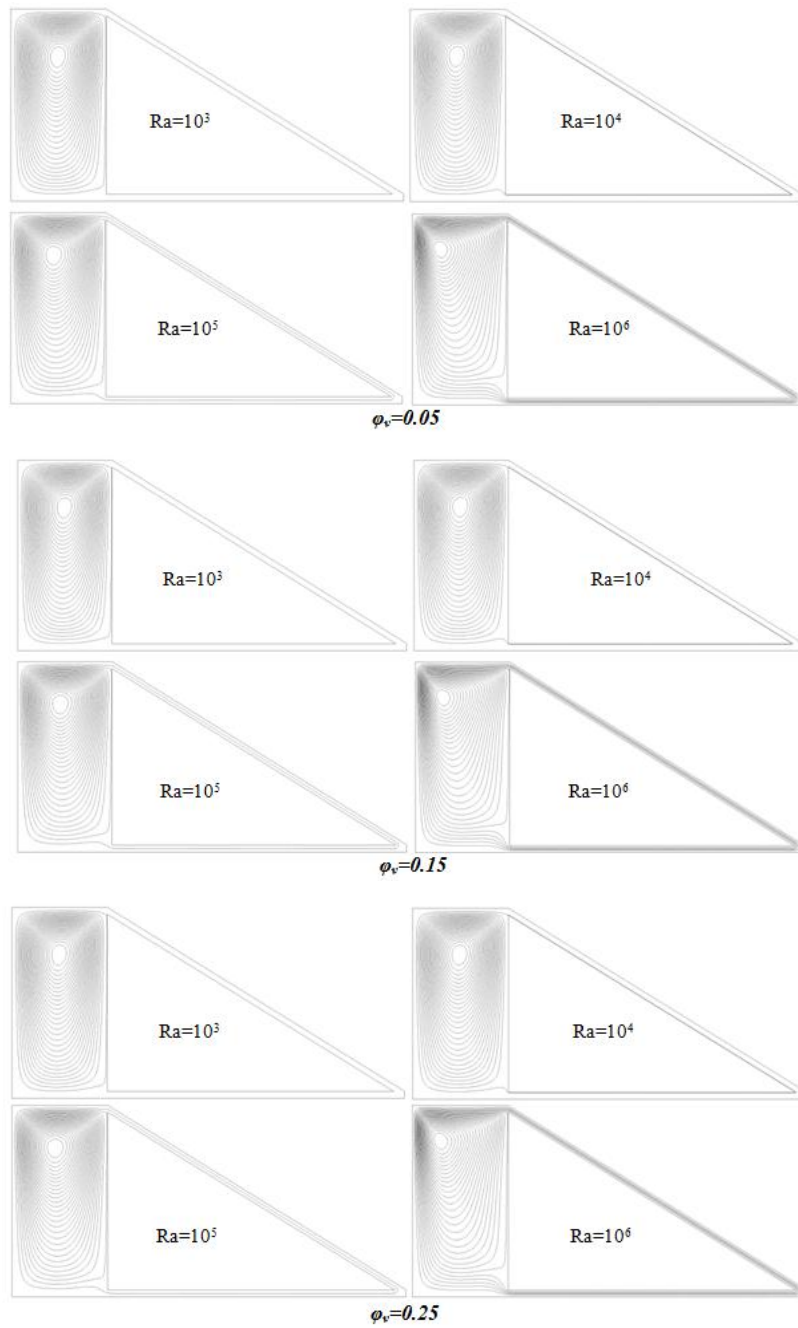


Figure 6 : Streamlines

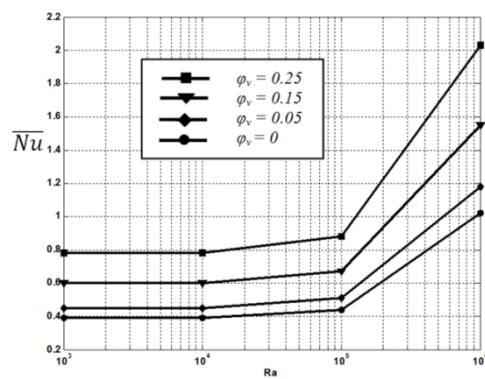


Figure 7 : Variation the average Nusselt number at the hot wall as a function of Ra for different values of  $\phi_v$ .



## 5. Conclusions

In this numerical study we modeled the heat transfer by laminar natural convection in a solar thermal water heater enclosure. The purpose of this study is the evaluation of the improvement of heat transfer which occurs when a water-copper nanofluid is used, as compared to a pure fluid (water). The results obtained clearly show that the use of nanofluids can influence greatly the heat transfer in the enclosure. The use of a water-copper nanofluid as coolant with volume fraction equal to 0.25 increases the heat transfer by a 100% as compared to the use of pure water. Also for all the values of Ra and  $\phi_v$  we noticed the formation of a rotating cell in the thermal storage tank. The intensity of recirculation of this cell becomes greater when increasing the value of Ra. Increasing the velocity of recirculation also implies improved heat transfer. The results also show that the case of  $\phi_v = 0.25$  corresponds to the best value of the volume fraction of the nanofluid water-copper at which the maximum heat is transferred through the hot wall in this solar thermal water heater enclosure.

## References

1. PATANKAR, S. V., Numerical heat transfer and fluid flow, Hemisphere publishing corporation, États-Unis of Americus, (1980).
2. X.W. WANG, X.F. XU, S.U.S. CHOI: Thermal conductivity of nanoparticle-fluid mixture, J. Thermophys. Heat Transfer, 13(1999), 474-480.
3. S.K. DAS, N. PUTRA, P. THIESEN, W. ROETZEL: Temperature dependence of thermal conductivity enhancement for nanofluids, Journal of Heat Transfer 125 (2003), 567-574.
4. K. KHANAFER, K. VAFAI, M. LIGHTSTONE: Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluid, Int. J. Heat Mass Transfer, 46 (2003), 3639-3653.
5. Hakan F. Oztop et Eiyad Abu-Nada. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. International Journal of Heat and Fluid Flow 29 (2008) 1326–1336
6. M. A. Mansour, A. Y. Bakier, M. A. Y. Bakier: Natural convection of the localized heat sources of T-shaped nanofluid-filled enclosures, American Journal of Engineering Research (AJER), 02 (2013), 49-61
7. Gladés Bachir: Contribution à l'étude de la convection naturelle dans les nanofluides en configuration de Rayleigh-Bénard, Doctoral thesis, University of Toulouse III- Paul Sabatier, France
8. Felix A. Peuser, Karl-Heinz Remmers, Martin Schnauss, Installations solaires Thermique conception et mise en œuvre, Observ'ER France (2005).