Thermal development for a pseudoplastic fluid in simple duct with consideration of viscous dissipation

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Abstract: In this work, we treat the thermal development problem, for a pseudoplastic fluid in a single pipe. A fully developed flow is supposed at the pipe inlet, with an imposed temperature at the surface in the case of a heating and cooling. In addition, the effect of viscous dissipation is considered. Finite difference method with an implicit scheme is used to solve the energy equation.

The main objective of the work is to provide results, enabling to well understand the effect of viscous dissipation associated with that of rheological behavior. For this, different values of the Brinkman (*Br*) number characterizing the heat generation by viscous friction, and the rheological index (*n*) have been taken for in heating situation as well as cooling. It has been found that the fluid shear-thinning (***n***↓) significantly reduces the dissipative effect, by reducing the friction between the fluid layers.

Keywords: Forced convection; thermal development; pseudoplastic fluid; viscous dissipation.

***Nomenclature*:**

r : Radial coordinate (*m*),
z : Axial coordinate (*m*),
T: Temperature (°*C*),
u: Axial Velocity ($m.s^{-1}$),
m: Fluid consistency ($Pa.s^n$),
n: Rheological index,
R: Dimensionless radial coordinate,
Z: Dimensionless axial coordinate,
θ: Dimensionless temperature,
Br: Brinkman number,
Pe: Peclet number,

***Subscript*:**

m: Mean (bulk for temperature),
W: Wall,

1. Introduction:

In a viscous fluid flow, there is always a friction between the fluid layers (shear). In some cases, this friction is considerable and provides a generation of significant heat within the flow. This heat generation is considered an internal heat source, which changes the temperature distribution in the medium and thus, the coefficient of heat transfer (*Nu*). The effect of viscous dissipation is usually represented by the Brinkman number (*Br*), which also depends on the heating conditions.

Today, viscous dissipation is very exploited in industry. One can cite as an indication plastic polymers extrusion. Originally powder; they are introduced into the extruder, equipped with a screw (one or more). By turning, a highly significant shear is applied to the polymer (particles of powder) which begins to collapse under the effect of heat generated by viscous dissipation. Slowly along the extruder, the temperature increases in the polymer and reaches that of its fusion. Thereafter the molten polymer passes to the molding phase.

From the above, the Graetz problem taking into account the viscous dissipation interest many authors. Among them, R. M. Cotta et al [1] studied the Graetz problem in a simple cylindrical pipe and between two parallel plates, for an imposed heat flux. The fluid was considered non-Newtonian, modeled by the power-law. The main study objective is to determine the Nusselt number. Three values of the rheological index were taken (n=1/3, n=1.0, n=3.0). A comparison with the asymptotic solution of Bird et al [2] shows a good agreement. For a cylindrical pipe, A. Barletta [3] studied the problem for three heating types: A decreasing flux until zero value for an infinite length, a decreasing flux until a positive value, and then an increasing to infinity flux when length tends to infinite. The author found that the value of the Nusselt tends to zero for the first case. It depends on the value of n and the reference Brinkman number (Br_∞) for the second case. And depends on n and a new dimensionless number β to the third case. H. Ragueb et al [4] have studied the above problem but in an elliptical section with imposed temperature. Numerical solution based on DADI scheme (Dynamic Alternating Direction Implicit) is followed. The Nusselt number is represented for different values of *Br* and β (ellipse radii ratio=a/b). They showed that the value of fully developed Nu does not depend in *Br*, but it increases with β. O. Aydin [5] analyzed for a developed dynamic regime in a cylindrical pipe and a Newtonian fluid, the effect of viscous dissipation on the temperature profile, and this for two heating conditions: A constant flux (CHF) and a constant temperature (CWT). The radial distribution of temperature and Nusselt number were obtained for different values of *Br*. An attractive analytical development made by the author for the case of an imposed flux leading to the following expression of the temperature profile:

$$\theta(R) = (1 - 4R^2/3 + R^4/3) - Br(R^2 - R^4)/3$$

For the second condition, a work similar to that done in [6; Chapter 4] is followed. In a subsequent study, the author [7] expands his work by considering the cooling case (Br<0.0). Different graphical presentations of the radial and axial temperature distribution for different values of *Br* are carried out. Nusselt values are also presented for both heating cases and for different values of *Br*. T. Basu et al [8] have investigated analytically the Graetz problem of a Newtonian fluid flowing in a circular pipe. The wall is subjected to a uniform temperature or a uniform flux. They presented simple analytical formulas for calculating the Nusselt number according to *Br*. The results are also presented for both heating conditions. Having noticed a singularity (Leap from $-\infty$ to $+\infty$) in the local Nusselt evolution when the case of cooling is considered (Br <0.0). E. Magyari et al [9] studied the problem by taking a non-uniform temperature profile at the entrance. Some manipulations of the energy equation in order to have an analytical expression profile are done. The authors considered both heating and cooling cases. They showed that for Z=0.00234, *Nu* coincides with Nu_∞. Graphical results $Nu(Z)$, for the case of uniform and non-uniform inlet temperature are given, indicating that the singularity disappears in the range -6/5 ≤ Br≤ 0.0. For two different geometries (cylindrical pipe and two parallel planes), P. M. Coelho et al [10] analyzed the Graetz problem in the presence of viscous dissipation for a viscoelastic fluid using the separation of variables method. Imposed temperature or flux are supposed as conditions at the walls. The authors show the *Nu* evolution versus *Z* change for different values of *Br* and *We*. A remarkable improvement of the *Nu* (better heat exchange) with increasing Webber number (*We*) considered for all *Br* is observed. This improvement is accompanied by a reduction of the thermal length which reduces the size of the heat exchanger.

The literature analysis, reveals that the effect of rheological index (*n*) of the pseudoplastic fluid was not properly disclosed. In addition, the majority of reviewed works interested in Nusselt established or in its evolution. It is noted that for a better understanding of the effect of viscous dissipation in this case of fluids, it is appreciated to consult some works where it is neglected. A good bibliographical summary done in [11] ensures this.

In this work, we discuss the effect of viscous dissipation and rheological index of the mixture temperature and Nusselt. An imposed temperature is supposed as thermal condition. Both heating and cooling cases are studied.

2. Problem formulation:

The geometry considered is a simple cylindrical pipe (Figure.1). The equations are written in the cylindrical coordinate system.

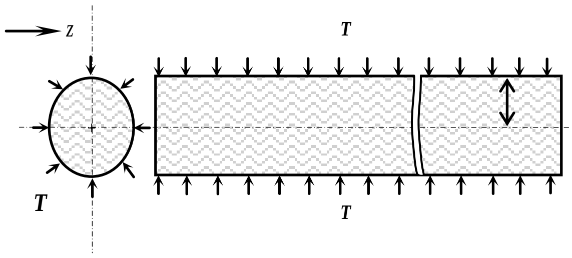

Fig.1 : Pipe geometry

2.1. Assumptions:

- Fully developed flow at the inlet;
- Axial diffusion neglected;
- Pseudoplastic fluid described with Ostwald-de Waele law $\tau = m\dot{\gamma}^n$ (*n*≤1.0) [2];
- Constants physical proprieties ($\rho; k; m; C_p$).

2.2. Velocity Profile:

The dynamic flow is assumed developed at the entrance of the pipe. Its expression is given in equation (1).

$$\frac{u}{u_m} = \frac{3n+1}{n+1}\left(1-\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}}\right) \quad (1)$$

The detailed demonstration can be found in [11, 12].

In figure (2), we have presented the effect of the rheological index *n* on the dimensionless axial velocity profile. This effect results in an increasing velocity magnitude in the pipe axis as ***n*** increases and therefore a decreasing parietal velocity gradient. For small *n* (strongly shear-thinning fluids: ***n***<0.5), the shape becomes flat with a hardly identified maximum, where an even larger central area is observed as *n* is small. The interpretation of this effect is the increase of the parietal velocity gradient with decreasing *n*. This is due to the decrease of the fluid apparent viscosity ($\mu_a = m.\dot{\gamma}^{n-1}$) near the wall. The flow thus moves more freely, and a greater uniformity in the distribution of the velocity is obtained (flattened shape). It should be noted that for *n*≈0.0, the shape of the velocity profile is almost flat with constant amplitude along *r*, this is a reminder to the case of gas characterized by very low viscosities, the speed is characterized by a mean value (flow/Section).

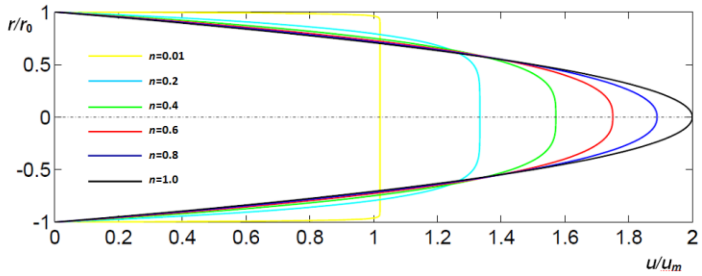

*Fig. 2: Effect of **n** on the velocity profile*

2.3. Energy equation:

The dimensionless velocity profile expression is injected into the energy equation below, in which the last term on the right describes the viscous dissipation:

$$\rho C_p u \frac{\partial T}{\partial z} = k \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \tau_{rz}\frac{\partial u}{\partial r} \qquad (2)$$

Where: k, ρ *and* C_p are thermal conductivity, density and specific heat respectively.

We replace *u* and τ_{rz} by their expressions, we obtain:

$$u_m\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial T}{\partial z}\right) = \frac{k}{\rho C_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{m}{\rho C_p}\left(\frac{u_m}{D}\right)^{n+1}\left(\frac{6n+2}{n}\right)^{n+1}\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}} \qquad (3)$$

This equation governs the radial and axial temperature changes.

To facilitate the numerical solution of the energy equation, the following dimensionless parameters are introduced:

$$R = \frac{r}{D}\ ;\ Z = \frac{z}{D}\ ;\ U = \frac{u}{u_m}\ ;\ \theta_{(T\,imp)} = \frac{T_w - T}{T_w - T_e}$$

Used in equation (3), we get the following dimensionless equation:

$$\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{R}{0.5}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial \theta}{\partial Z}\right) = \frac{1}{Pe}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right) - \frac{Br}{Pe}\left(\frac{6n+2}{n}\right)^{n+1}\left(\frac{R}{0.5}\right)^{\frac{n+1}{n}} \qquad (4)$$

Where: $Pe = \frac{u_m.\rho.C_p.D}{k}$ (Peclet number); $Br = \frac{m u_m^{n+1}}{k D^{n-1}(T_w - T_e)}$ (Brinkman number)

The last dimensionless number appears after nondimensionalization, is the source term due to viscous dissipation. This heating phenomenon by viscous friction will be more pronounced when the *Br* value is high and vice versa.

Boundary conditions:

$$
\begin{aligned}
&Z=0 \quad \rightarrow \ \theta=1.0 \\
&R=0 \quad \rightarrow \ \partial\theta/\partial R\big|_{R=0.5}=0 \\
&R=0.5 \rightarrow \theta=0.0
\end{aligned}
\tag{5}
$$

3. Numerical Resolution:

The finite difference method is chosen to discretize the terms following *R* and *Z* directions. Simple implicit scheme is then followed for the rearrangement of discretized terms to make the use of the Thomas algorithm possible. A regular grid in *R* direction is selected while a gradually growing mesh following *Z* is adopted. For our case, and after the convergence study (Figure (3)) we took $\Delta R=10^{-3}$ (501 Nodes), whereas following *Z*, we took $\Delta Z_0=10^{-7}$ with an amplification factor of 1.005 until a maximum ΔZ equal to 10^{-3}.

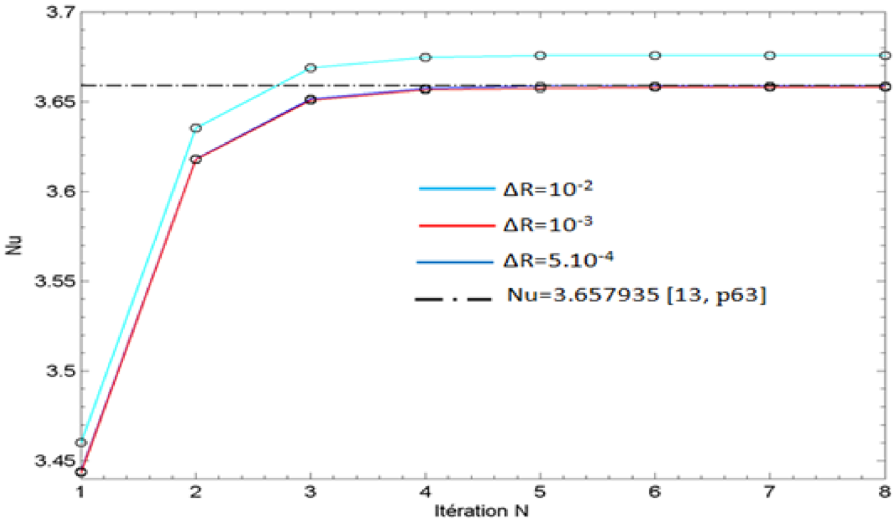

Fig.3: Radial grid effect on the value of Nu_{fd} and the necessary iterations number for convergence Br=0.0

3.1. Code validation:

In order to make the present work valuable, two validations were made. The first on the evolution of the Nusselt number along the pipe by neglecting the effect of viscous dissipation with Pe=1.0. Our results compared to the case of a Newtonian fluid with those of R.M. Cotta and al [1] for two heating types (Table.1). a very good agreement between the two works is observed.

A second validation is done for the value of Nu established for different values of *n*. Our results for specified *n* were compared with those of H. Ragueb et al [4] for the case of temperature imposed, and those of A. Barletta [3] for an imposed flux case. Our results show a good accuracy with literature (Table.2).

Table 1: Evolution of Nu along Z. n=1.0; Br=0

Z	Imposed flux		Imposed Temperature	
	This work	Ref [1]	This work	Ref [1]
0,0001	27,30328	27,2760	22,30369	22,279
0,0005	15,82263	15,8130	12,83301	12,824
0,001	12,54433	12,5380	10,13574	10,13
0,005	7,498165	7,4937	6,00512	6,0015
0,01	6,15188	6,1481	4,91886	4,9161
0,02	5,20167	5,1984	4,17471	4,1724
0,05	4,51721	3,5139	3,71199	3,71
0,1	4,37786	3,3748	3,65981	3,6581
0,2	4,36912	3,3637	3,65842	3,6568

Table 2: Values of established Nu for different n

n	Imposed Temperature		Imposed Flux	
	This work	Ref [4]	This work	Ref [3]
0,2	17,71237	17,2681	3,44943	3,4474
0,5	11,64363	11,5239	2,4559	2,44881
1	9,58713	9,5225	1,37522	1,3714
2	8,54769	8,5019	0,30672	0,3026
5	7,91391	7,8818	0,00152	0,0015

4. Results and discussion:

View of the many parameters involved in the problem (*Br*, ***n***, heating condition), the results will be presented for the case of heating then for the case of cooling.

Generally, the generation of heat due to viscous dissipation is characterized by a gradual increase of the temperature within the fluid. This growing increase with increasing *Br*, manages to bring the fluid temperature to levels sometimes exceeding the temperature of the wall. This means that fluid starts from an axial position to heat the wall. Details on this point with many results are given in [6, Chap 3]. The referral to this reference is the fact that its presentation requires many figures which the paper space limitation prevents.

An important element in the heat transfer problems is the bulk temperature. In addition it goes directly into the calculation of the Nusselt, it serves as an evaluation gauge the overall evolution of the fluid temperature. This of the fact that is a weighted average (by the velocity distribution**),** which gives an idea about the thermization process (or cooling).

In the present work, the dimensionless bulk temperature is calculated from equation (6):

$$\theta_m = 8 \times \int_0^{1/2} U \times \theta \times R \times dR \qquad (6)$$

We note that a trapezoidal integration rule is used to calculate the integral.

In the following, we will present the axial evolution of the bulk temperature, from the inlet to the fully developed stage. Two values of *n* are chosen (*n*=0.1 and *n*=1.0), for five values of *Br*. This is done to the case of a heating and the cooling. We note that for the case of cooling, *Br* is taken negative simply (see his expression). In addition, the Peclet number is taken equal 100.0, to neglect the axial conduction.

4.1. Evolution of the bulk temperature:

a. Heating at the wall:

The case of heating at the wall is shown in Figure (4). The dotted line is for $\theta_W = 0.0$. It is clear that the dimensionless temperature becomes increasingly negative when ***Br*** increases, indicating a strong heating in particular in the central area due to the generation of heat by viscous friction. The rheological index also affect this evolution, where high heat is observed for ***n***=1.0 compared to ***n***=0.1. This is explained by the fact that the decrease of ***n*** results in a decrease in the viscosity and therefore the viscous friction directly related.

From the two subfigures (***n***=0.1 and ***n***=1.0), we can see approximately, the axial position from which the heat transfer changes direction. This position becomes closer to the inlet when **Br** and/or ***n*** increase.

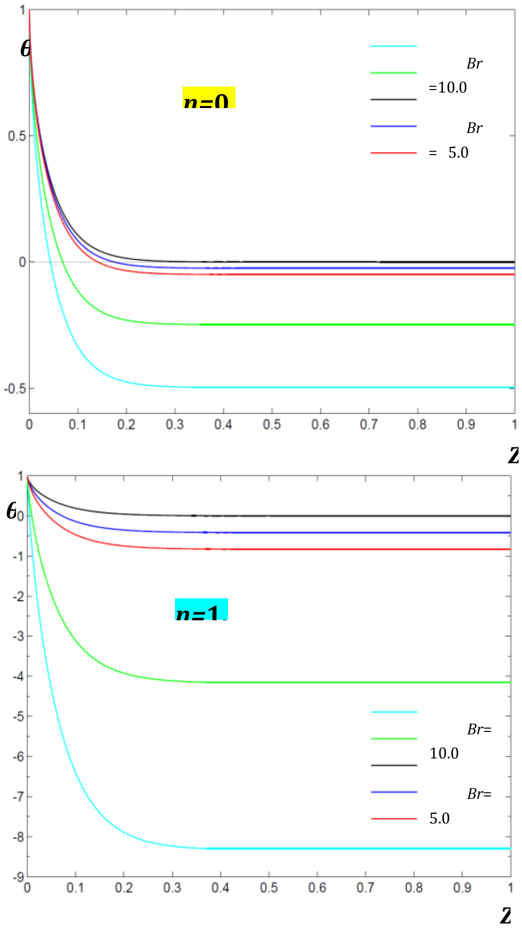

Fig. 4: Evolution of θ_m along the pipe for different n and Br. Heating

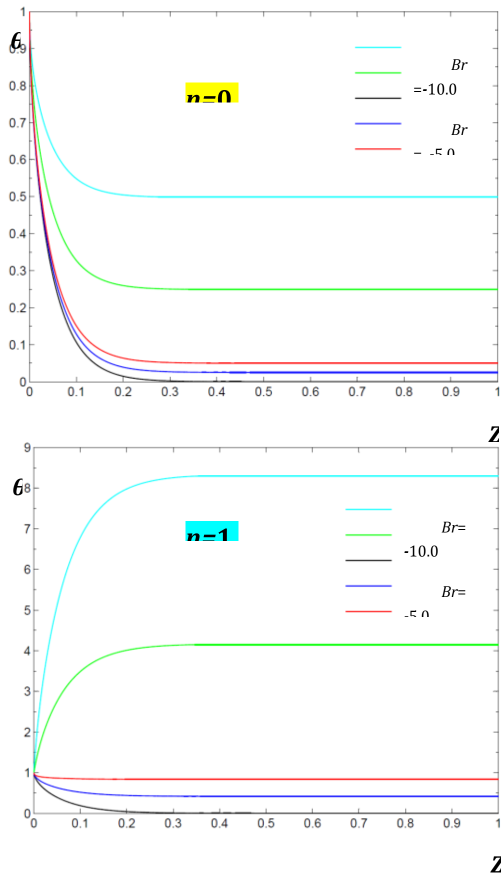

Fig. 5: Evolution of θ_m along the

Cooling at the wall:

Contrary to the previous case, the bulk temperature in this case increases with the increase of *Br*. The values of the bulk temperature for *n*=1.0 are larger than those corresponding to *n*=0.1 (Fig.5).

A new phenomenon is observed for *Br*= -5.0 and *Br* = -10.0 for *n*=1.0. The bulk temperature changes slope from the entrance, where very high values are recorded reaching 8.0, while for other cases it remains below 1.0. This in our opinion is explained by the fact that the great value of Br associated with high *n*, leads to strong heating by viscous friction. This friction intensively grows the dimensionless temperature even before the cooling process begins (ie. just at the entrance).

This phenomenon is not observed for *n*=0.1, the fact that the shear-thinning reduces the friction between the fluid layers. It is clear that there is a value of *n* from which we begin to see this phenomenon.

4.2. Evolution of the Nusselt number:

Nusselt number, which characterizes the intensity of the convective exchange, is another important parameter. The effects of the rheological index *n* and the Brinkman number *Br* are presented in figure (6) for heating case, and in figure (7) for the cooling one.

We note that we are limited to *n*=0.1 and *n*=1.0, for lack of space, other values are plotted in [6, Chap 3].

a. Heating **of** the wall**:**

In this part the Nusselt evolution was presented for *n*=0.1 and *n*=1.0 and five *Br* values (Figure 6). The black line is for the case *Br*=0.0 (negligible viscous dissipation), where the habitual curves are found and the value of *Nu* in the establishment is mentioned. But when *Br* is not zero, we see an evolution that seems strange for the two sub-figures. Indicating that the rheological index *n* and *Br*, do not affect the evolution shape but affect the *Nu* value.

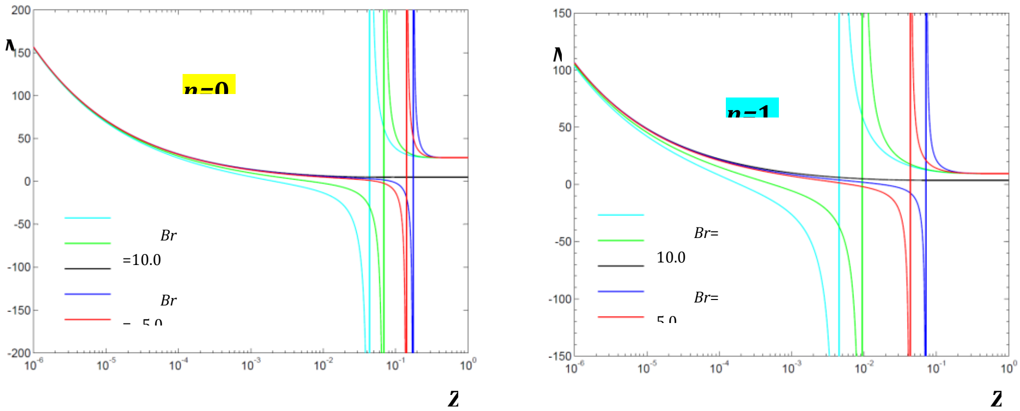

Fig.6: Evolution of Nu along the pipe for different ***n*** *and* ***Br****.*

The Nusselt decreases gradually from the inlet. But it continues to decrease to very low negative values, and then at an axial position, it abruptly rises where a jump is recorded. Finally, it goes down to its establishment value.

The explanation for this change is as follows: at the entrance, the fluid temperature and that of bulk are very close to the inlet temperature (θ_e). This will give a negative term in the numerator close to 1.0 and a negative term close to 1.0 multiplied by ΔR in the denominator ($Nu = +\frac{1}{\theta_m}\partial\theta/\partial R|_{R=0.5}$).This will give a great positive *Nu*. Away from the inlet, the fluid may be hot, and under the effect of the viscous dissipation its temperature exceeds that of the wall, negative values are obtained, recalling that θ_m is always lower than that of the wall. After some distance, θ_m becomes equal to that of the wall (=0.0), and the jump in the value of *Nu* is observed, its value becomes positive and a decrease until the establishment was observed thereafter. By increasing, *Br* moves the point of singularity to the inlet since it intensifies the heat generation by viscous dissipation, which accelerates the phenomena described above. The rheological engenders the opposite effect.

The analysis of the sub-figures shows that Nusselt curves met at the same value at the establishment, regardless of the value of *Br*. This is explained by the fact that its calculation is conditioned by θ_W fixed along the whole pipe length.

b. Cooling **of** the wall**:**

The case of cooling is now considered. The Nusselt numbers evolutions are plotted in Figure (7) for the same values of *n* considered in the previous case, and five negative *Br*. It is clearly seen that *Nu* curves for *n*=0.1 have minimums, followed by lift until establishment. This is valid for all *Br* supposed. It is explained by the fact that θ_{N-1} (close to wall) decreases faster than θ_m near the entrance. This process continues until reaching the *Nu* minimum. With the generation of heat by viscous dissipation θ_m also rapidly decreases and a lift in *Nu* shape is obtained.

The rheological index ***n*** has a remarkable effect on the evolution of *Nu*, as its increase favors the viscous dissipation effect as detailed above. This can be observed for the curves of *Br*=-0.5 and *Br*=-1.0, where a similar behavior to that observed for *n*=0.1 is obtained. Contrary for the other *Br* and in particular for the strong *n*, Nusselt does not exhibit a minimum. This is the cause of the strong disturbance of thermal field by viscous heat generation, which grow rapidly the bulk temperature and hence the Nusselt closer to the entrance.

The same phenomenon at the establishment stage is reproduced for the case of cooling, where the curves at different *Br* are joined at the same value.

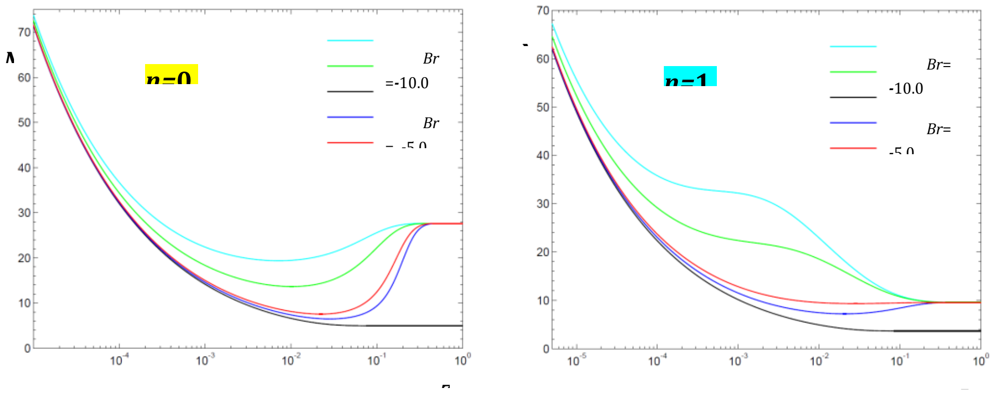

*Fig. 7: Evolution of Nu along the pipe for different **n** and **Br**. **Cooling***

5. Conclusions:

The results obtained from this Numerical work show that:

- The viscous dissipation increases with *Br*, and increasingly high temperatures are recorded;
- The decrease in rheological index *n* reduces the effect of viscous dissipation due to the reduction of friction between the fluid layers;
- A singularity in the Nusselt evolution is observed in the case of heating. This singularity becomes closer to the entrance when *Br* and/or *n* increase.

  In the case of cooling, the Nusselt number increases near the entrance with the increase of *Br*. Its evolution is characterized with a slope down followed with a lift when viscous dissipation becomes intense. This result is also affected by *Br* an *n* values;
- For the two cases supposed, the fully developed Nu value is independent to the change of ***Br***. But it is greater of that for *Br*=0.0.

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