Stress Concentrations in Composite Plates With a Circular Hole

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Résumé - Le Perçage des trous sur les matériaux composites est nécessaire pour les besoins mécaniques comme la diminution le moment des inerties des disques des machines tournants et pour l'assemblage des pièces mécanique et utilisé pour l'évacuation la chaleur dans les systèmes de refroidissements …etc. La préséance des trous dans cette structures composites est provoques du phénomène de concentration des contraintes. Cette phénomène que influe négativement sur leur comportement mécanique. Dans cette étude nous déterminer l'influence de la présence du trou circulaire sur le comportement mécanique des plaques orthotropes. En utilisé deux approches des théories sont la théorie de Lekhnitskii (1968) et Green-Zerna (1954). La variation du facteur de concentration a été étudiée en fonction du différent d'angle d'orientation des fibres. Alors l'analyse d'élément finis a été employée pour calculer cette variation.

Mots Clés : Matériau composite. Plaques perforé. Concentration des contraintes. Trou circulaire. Facteur de concentration des contraintes. Orientation des fibres.

Keywords: Composite material, Perforated plates, Stress concentration, Circular Hole, Stress concentration factor, Fiber orientation.

1. Introduction

Use of composite materials with an organic matrix in particular laminates. Continues to grow in the most varied fields. Good specific mechanical properties of these materials make it possible a lighter structures. Highly sought after in mechanical constructions of transport. The assembly of composite elements to a structure often requires the production of holes for receiving bolts or rivets or the passage of electrical and phone wires, for various practical reasons, the presence of a hole leads to a weakening of the structure due to local stress on the known stress concentration. For plates orthotropic the value of the stress concentration factor can be different from that recorded for isotropic plates is worth 3. Indeed this factor varies depending on the fiber orientation. It reaches the maximum value when the fiber orientation is parallel to the pulling direction ($\theta = 0°$), and minimal for the fiber orientation perpendicular to the tensile direction ($\theta = 90°$) [7]. Several research studies have investigated the case of rectangular composite plates with circular hole under the simple tension and compression, the theory of Lekhnitskii [2] is approached chosen to analyze the stress distribution in an orthotropic plate with holes is the one proposed in (1968). And the Theory of Green-Zarna [3] approached chosen to determine the stress distribution around circular hole in a composite plate is the one proposed in (1954). The analytical and numerical study is based on approximate solutions of the stress field and the numerical calculation using the finite element method. The works of R. Boubeker and M. Hecini [7] located on the effect of the presence of circular hole on the mechanical behavior of orthotropic plates. Kaltakci [8] study the concentration of forced perforated plates in laminated materials used in the analytical approximate solutions Green -Zerna for the purpose of presenting the instrument fiber orientation layers on the change factor concentrate stress to the circular hole edge. The work of references [10-12] based on the resolution of Lekhnitskii complex variable method equations to determine the stress field at the edge of differing geometric singularity and determine the effect of biaxial loading on the change stress concentration factor. H Murat Arslan [9] presented the effect of anisotropy ratio E_1/E_2 to the stress concentrations factor in voicing circular hole centered in the rectangular multilayer The study shows that the increase of the anisotropy ratio E_1/E_2 actually increase the stress concentration factor for the fiber orientation of 0°, and decrease for the 90° orientation of the fibers. S, C, Tan, R, Y, Kim [13] using good condition hovels experimentally using strain gauges to determine the stress concentration of orthotropic laminated contain a circular hole subjected to tensile loading model is orthotropic or anisotropic plate with finite dimensions by a contribution of hole size. Good correlation was obtained between theory Tan S, C, (1988) and the experimental results. The work of references [11-15] based on the analysis of displacements or deformations in experimental huts used modern techniques for analyzing the behavior of perforated plates from composite materials. The advantage of these techniques is a fact that the correlation system does not perform the measurement in a single point as in case of conventional measures such gauges.

These studies show that the stresses are not homogeneous in the vicinity of the hole and that this stress field can be considered homogeneous from the hole.

1.1. Stress Concentration Factor in isotropic materials

The thickness *h* of the plate with a circular hole countered with radius *R*, Apply the pure tensile stress in dirction of axis 1 to the plate, the dimensions of the plate are assumed sufficiently large compared to the radius R, and the volume forces are negligible in plane stress [1].

The analytical solution of the plane elasticity distribution of stress field in polar coordinates (0, r, α) is identified by [1]:

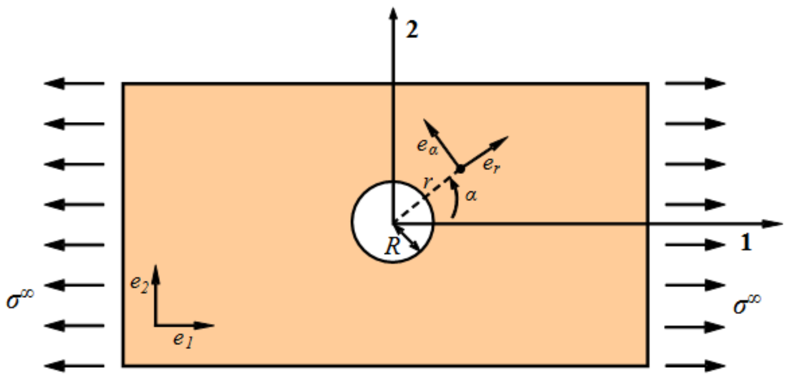

Figure 1 : Plate isotropic with a hole to the simple tensile [1]

$$\frac{\sigma_{rr}}{\sigma^\infty} = \frac{1}{2}\left[\left(1-\frac{4}{\rho^2}+\frac{3}{\rho^4}\right)\cos 2\alpha + 1 - \frac{1}{\rho^2}\right] \tag{1}$$

$$\frac{\sigma_{\alpha\alpha}}{\sigma^\infty} = \frac{1}{2}\left[-\left(1+\frac{3}{\rho^4}\right)\cos 2\alpha + 1 + \frac{1}{\rho^2}\right] \tag{2}$$

$$\frac{\sigma_{r\alpha}}{\sigma^\infty} = -\frac{1}{2}\left[\left(1+\frac{2}{\rho^2}-\frac{3}{\rho^4}\right)\sin 2\alpha\right] \tag{3}$$

With $\rho = r/R$

The stress concentration factor defined by the ratio of the maximum stress and nominal strain σ^∞ apply away from the hole:

$$K_t = \frac{\sigma^{\max}}{\sigma^{\infty}} \tag{4}$$

In this case $K_t = 3$

In particular, the edge of the hole ($r = R$):

$$\begin{cases} \sigma_{rr}(r = R) = 0 \\ \sigma_{\alpha\alpha}(r = R, \alpha) = \sigma\,(1 - 2\cos(2\alpha)) \\ \sigma_{r\alpha}(r = R, \alpha) = 0 \end{cases} \tag{5}$$

The orthoradial stress is:

$$\sigma_{\alpha\alpha}(r = R) = \sigma(1 - 2\cos(2\alpha)) \Rightarrow \frac{\sigma_{\alpha\alpha}(r = R)}{\sigma^{\infty}} = 1 - 2\cos(2\alpha) \tag{7}$$

1.2. The finite element Modeling and simulation

The structural calculation software ANSYS finite element method was used to analyze the stress concentration problem in the plate with a hole (Fig. 1). Geometry considered is meshed by PLANE 82 rectangular 2D element. This element has eight nodes and six degrees of freedom (three translations and three rotations). According to the study of the convergence element size was adapted to 1 mm. The conduction boundary and the applied loads are similar to a tensile test and pure compression. The analysis has to determine the stress field in the plate and using the once dimensional units to directly determine the value of the stress concentration factor in this field by dividing the nominal stress σ^{∞} [15].

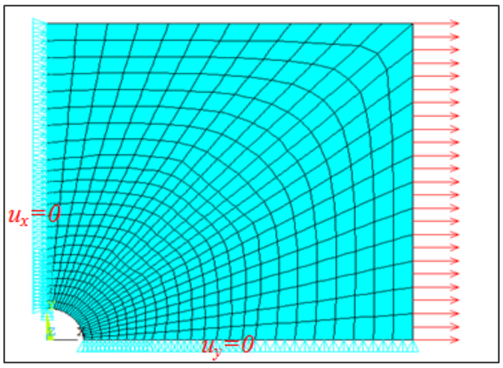

Figure 2: Model of finite elements of the plate with a hole

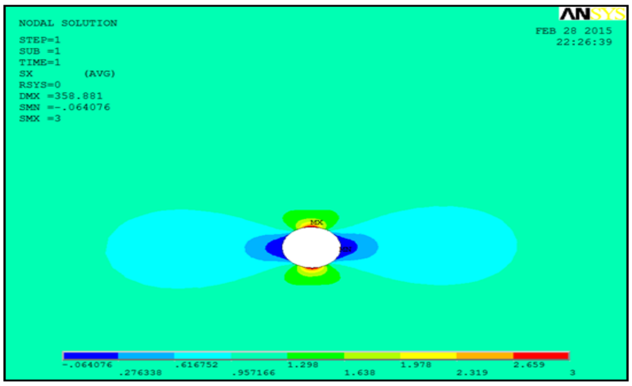

Figure 3: Distribution of the stress concentration factor the plate isotropic in tensile

2. Distribution of the stress in a plate orthotropic with a hole

In this study we will conduct an analysis of the stress concentration around and away from a circular hole in an orthotropic plate. This study will be based on approximate solutions of Lekhnitskii theory and Green-Zerna Theory and numerical calculated by the finite element method.

2.1. Theory of Lekhnitskii

The theory Lekhnitskii [2] approached chosen to analyze the stress distribution in a plate orthotropic with a hole is that proposed by Lekhnitskii in (1968). An orthotropic plate containing a circular hole of radius R centered subjected to simple traction σ^{∞} nominal stress acting at an angle φ by contribution to the elastic principal axis of the plate 1, The dimensions of the plate are assumed sufficiently large compared the radius of hole (Fig. 4), Volume forces are negligible. We assume a plane strain state.

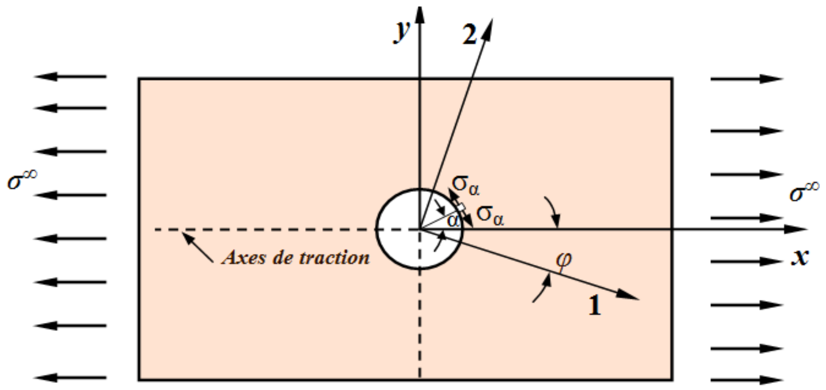

Figure 4: Tensile the elastic orthotropic plate with a circular hole

Next the Lekhnitskii theory [2] the stress concentration factor can be expressed by the following formula:

$$K_t = \frac{E_\alpha}{E_x}\left\{\begin{matrix}[-\cos^2\phi + (m+n)\sin^2\phi]m\cos^2\alpha + [(1+n)\cos^2\phi - m\sin^2\phi] \\ \sin^2\alpha - n(1+m+n)\sin\phi\cos\phi\sin\alpha\cos\alpha\end{matrix}\right\} \qquad (8)$$

E_α: is the modulus of elasticity in the direction of α given by the following equation (Fig. 4):

$$\frac{E_\alpha}{E_x} = 1\Big/\left[\sin^4\alpha + \frac{E_x}{E_y}\cos^4\alpha + \frac{1}{4}\left(\frac{E_x}{G_{xy}} - 2\nu_{xy}\right)\sin^2 2\alpha\right] \qquad (9)$$

φ (° /): Application Angle of the tensile force measured by the input axis 1 (Fig. 4).

α (° /): Location angle of the stress concentration factor.

E_x, E_y, G_{xy}, ν_{xy}: are the elastic characteristics of the material.

The value of *m* and *n* are defined by:

$$m = \sqrt{\frac{E_x}{E_y}} \qquad (10)$$

$$n = \sqrt{2\left(\sqrt{\frac{E_x}{E_y}} - \nu_{xy}\right) + \frac{E_x}{G_{xy}}} \tag{11}$$

According Calcote [4] and Jones [5]. The elasticities characteristics E_x, E_y, G_{xy}, ν_{xy}, ν_{xy} in reference frame (x, y) are related to material elasticity modules E_1, E_2, G_{12}, ν_{12}, ν_{21} by the following equations:

$$\begin{cases} E_x = E_1 / \left[\cos^4\theta + \dfrac{E_1}{E_2}\sin^4\theta + \dfrac{1}{4}\left(\dfrac{E_1}{G_{12}} - 2\nu_{12}\right)\sin^2 2\theta \right] \\ E_y = E_1 / \left[\sin^4\theta + \dfrac{E_1}{E_2}\cos^4\theta + \dfrac{1}{4}\left(\dfrac{E_1}{G_{12}} - 2\nu_{12}\right)\sin^2 2\theta \right] \\ G_{xy} = E_1 / \left[1 + 2\nu_{12} + \dfrac{E_1}{E_2} - \left(1 + 2\nu_{12} + \dfrac{E_1}{E_2} - \dfrac{E_1}{G_{12}}\right)\cos^2 2\theta \right] \\ \nu_{xy} = \dfrac{E_x}{E_1}\left[\nu_{12} - \dfrac{1}{4}\left(1 + 2\nu_{12} + \dfrac{E_1}{E_2} - \dfrac{E_1}{G_{12}}\right)\sin^2 2\theta \right] \\ \nu_{yx} = \dfrac{E_y}{E_1}\left[\nu_{12} - \dfrac{1}{4}\left(1 + 2\nu_{12} + \dfrac{E_1}{E_2} - \dfrac{E_1}{G_{12}}\right)\sin^2 2\theta \right] \end{cases} \tag{12}$$

(12)

θ (°/) : Angle of the fiber orientation.

2.2 Theory of Green-Zerna

This approximate theory chosen to determine the stress distribution author circular hole in a composite plate is the one proposed by Green-Zarna in (1954) [3]. Where they proposed to study this problem using the following equation:

$$S_{22}\frac{\partial^4 F}{\partial x^4} + (S_{66} + 2S_{12})\frac{\partial^4 F}{\partial x^2 \partial y^2} + S_{11}\frac{\partial^4 F}{\partial y^4} = 0 \tag{13}$$

Where F is the Airy function S_{11}, S_{22}, S_{12}, and S_{66} are the coefficients of flexibility defined as follows:

$$S_{11} = \frac{1}{E_1},\ S_{22} = \frac{1}{E_2},\ S_{66} = \frac{1}{E_{12}},\ S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} \tag{14}$$

Equation (13) is developed by:

$$\left(\frac{\partial^2}{\partial x^2} + \beta_1\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2} + \beta_2\frac{\partial^2}{\partial y^2}\right)F = 0 \tag{15}$$

β_1 and β_2 and are positive real constant.

The stress concentration in orthotropic plates (Fig. 4) can be found using the equation of the tangential stress σ_α the edge of the hole.

The stress concentration factor can be expressed by the following formula [9]:

$$K_t = \frac{\sigma_\alpha}{\sigma^\infty} = \frac{N_1 + N_2 + N_3}{(1+\gamma_1^2 - 2\gamma_1 \cos 2(\alpha-\theta))(1+\gamma_2^2 - 2\gamma_2 \cos 2(\alpha-\theta))} \tag{16}$$

$$N_1 = (1+\gamma_1)(1+\gamma_2)(1+\gamma_1+\gamma_2-\gamma_1\gamma_2 - 2\cos 2(\alpha-\theta)) \tag{17}$$

$$N_2 = -4[\gamma_1 + \gamma_2 - (1+\gamma_1\gamma_2)\cos 2(\alpha-\theta)\sin^2\theta] \tag{18}$$

$$N_3 = -4(\gamma_1\gamma_2 - 1)\sin 2(\alpha-\theta)\sin\alpha\cos\alpha \tag{19}$$

σ^∞ *:* tensile stress or compression.

$$\gamma_1 = \frac{\sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{yx}\right) + \sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]}\right] - 1}}{\sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) + \sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]}\right] + 1}} \tag{20}$$

$$\gamma_2 = \frac{\sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{yx}\right) - \sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]}\right] - 1}}{\sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right) - \sqrt{\left[\left(\frac{E_2}{2G_{12}} - \nu_{21}\right)^2 - \frac{E_2}{E_1}\right]}\right] + 1}} \tag{21}$$

Where the values γ_1, γ_2 vary between -1 and 1. For isotropic plates, one can use the equations mentioned above to calculate the shear stress at the edge of the circular holes taking $\gamma_1 = \gamma_2 = 0$ [8].

3. Results and Discussion

3.1. Effect of fiber orientation angle on SCF

The orientation of the fibers is an important parameter that influée on change in mechanical prosperities of composite materials.

Table 1: Mechanical properties of differing unidirectional composites.

Material	E_1 (Gpa)	E_2 (Gpa)	G_{12} (Gpa)	ν_{12}	ν_{21}
Glass /epoxy A [6]	46	10	4,7	0,31	0,0673
Glass /epoxy B [10]	47,4	16,2	9,14	0,25	0,0889
Glass /epoxy C [10]	54,9	18,3	9,14	0,25	0,0833
Graphite /epoxy [10]	125	9,9	5,5	0,28	0,022

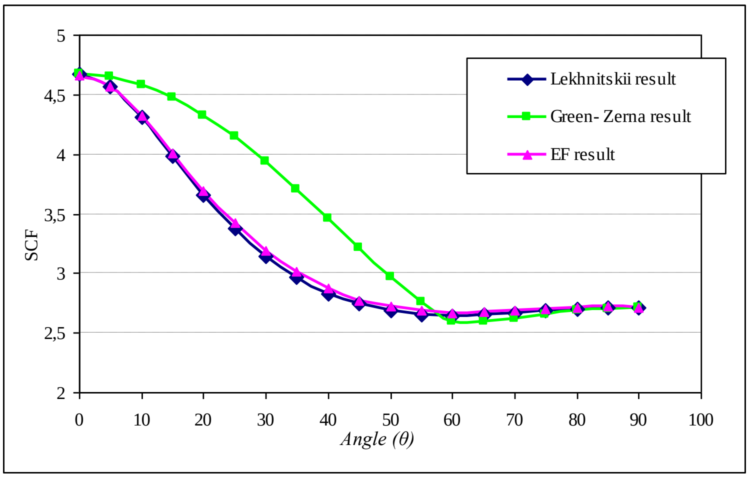

Figure 5: Variation of the stress concentration factor as a function of orientation of the fibers to the glass / epoxy A material tensile

The graph of figure 5 show the change of the stress concentration factor at the edge of a hole made on an orthotropic plate for glass / epoxy A material the elastic characteristics fates in Table 1, the numerical calculation is obtained by the finite element software ANSYS which the element used does not take into account the transverse shear in the thickness, the analysis shows that the maximum value of the tensile stress concentration factor is localized in the plate the orientation of fibers is 0°. it is 4,66. While the minimum value is in the plate to 60° and which is 2,6.

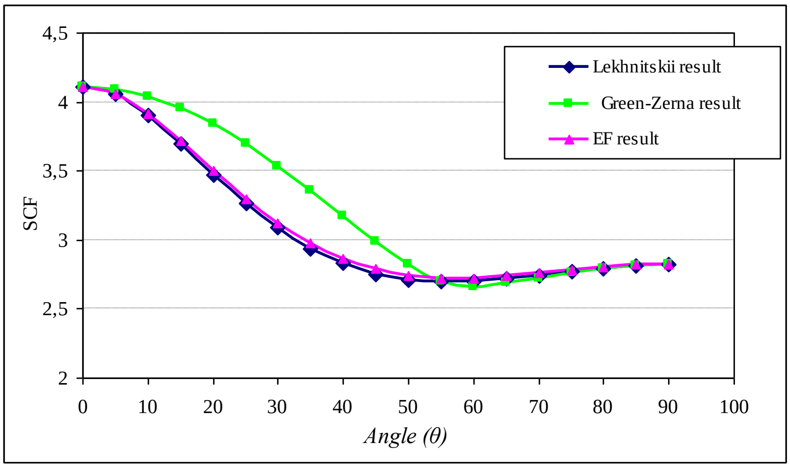

Figure 6: Variation of the stress concentration factor as a function of orientation of the fibers to the glass / epoxy B material tensile

Figure 6 show the variation of the stress concentration factor as a function of fiber orientations materials for glass / epoxy B. The study shows that the maximum value of the stress concentration factor in tension is localized in the plate whose fiber orientation is 0°. It is 4,1. While the minimum value is in the plate to 55° and which is 2,69.

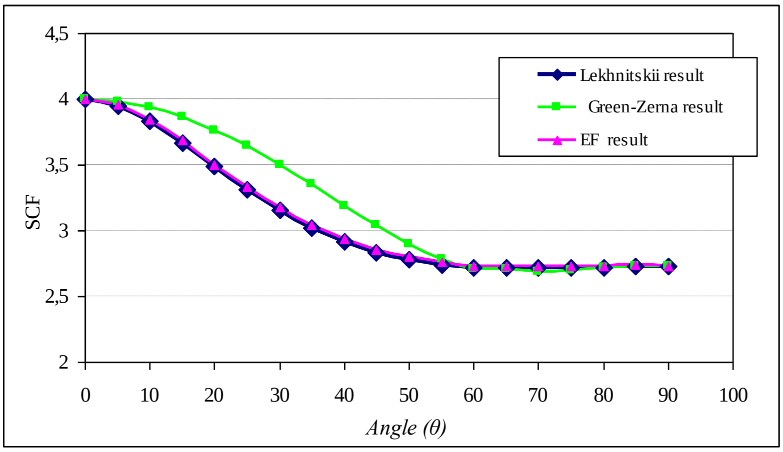

Figure 7: Variation of the stress concentration factor as a function of orientation of the fibers to the glass / epoxy C material tensile

The analysis shows that the maximum value of factor of stress concentration in tension is localized in the plate whose fiber orientation for glass / epoxy C material is to 0 °, it is 3,99. While the minimum value is located in the plate at 60° and 65° and which is 2,716. And the stress concentration factor taking the maximum compressive strain, the orientation of the fibers is 90 ° It is 1,732. While the minimum value is located in the plate at 0°, which is 0,577.

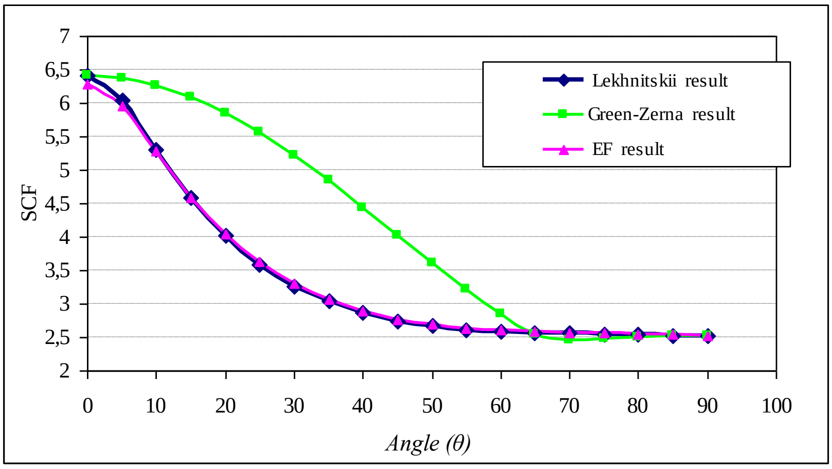

Figure 8: Variation of the concentration factor of the stresses in the fiber orientation according to the material Graphite /epoxy tensile

The study shows that the maximum value of the stress concentration factor in tension is localized in the plate, the fiber orientation for the graphite /epoxy material is at 0°. It is 6,4. While the minimum value is in the plate to 90° and which is 2,523.

4. Conclusion

For orthotropic plates the study shows that the value of the stress concentration factor can be different from that recorded for isotropic plates is worth 3 for traction. The study shows that the maximum values of the stress concentration factor is found in the α angle location in the position of 90° to the hole edge for tensile. Both theories have been used to determine the distribution of the stress concentration factor the circular hole edge for orthotropic plates but the study shows that for the study of behavior of orthotropic plate with circular hole. The use of the analytical method of Lekhnitskii theory is proposed, because their results are practically approaches to those given by the finite element method (ANSYS). For the fiber orientation (θ = 0°) (θ = 90°). The results of SCF given by both theories Lekhnitskii and Green-Zarna and the results given by the finite element method (ANSYS) are strictly identical. The study shows that the location of maximum and minimum values of the stress concentration factor is not influes depending on angle (α) of fiber orientation.

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